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Lyapunov-based Spacecraft Rendezvous Maneuvers using Differential Drag

D. Pérez[1] and R. Bevilacqua[2]
Rensselaer Polytechnic Institute, Troy, New York, 12180

This work presents a Lyapunov-based control strategy to perform spacecraft rendezvous maneuvers exploiting differential drag forces. The differential drag is a virtually propellant-free alternative to thrusters for generating control forces at low Earth orbits, by varying the aerodynamic drag experienced by different spacecraft, thus generating differential accelerations between the vehicles. The variation in the drag can be induced, for example, by closing or opening flat panels attached to the spacecraft, hence effectively modifying their cross- sectional area. In a first approximation, the relative control forces can be assumed to be of bang-off-bang nature. The proposed approach controls the nonlinear dynamics of spacecraft relative motion using differential drag on-off control, and by introducing a linear model. A control law, designed using Lyapunov principles, forces the spacecraft to track the given guidance. The interest towards this methodology comes from the decisive role that efficient and autonomous spacecraft rendezvous maneuvering will have in future space missions. In order to increase the efficiency and economic viability of such maneuvers, propellant consumption must be optimized. Employing the differential drag based methodology allows for virtually propellant-free control of the relative orbits, since the motion of the panels can be powered by solar energy. The results here presented represent a breakthrough with respect to previous achievements in differential drag based rendezvous.

[1] Ph. D. Student, Mechanical, Aerospace and Nuclear Engineering Department, 110 8th Street Troy, NY 12180; perezd3@rpi.edu USA. AIAA Student Member.
[2] Assistant Professor, Mechanical, Aerospace and Nuclear Engineering Department, 110 8th Street Troy, NY 12180; bevilr@rpi.edu. AIAA Member. www.riccardobevilacqua.com

Nomenclature

a_D = Relative acceleration caused by differential drag

$\mathbf{a}_{J2}$ = Relative acceleration caused by J_2

a,b,d = Constants in the transformation matrix

$\underline{\boldsymbol{A}}_{\boldsymbol{d}}$ = Linear guidance state space matrix

$\underline{\boldsymbol{A}}, \underline{\boldsymbol{B}}$ = Matrices for the state space representation of the Schweighart and Sedwick equations

c = Coefficient from the Schweighart and Sedwick equations

C_o = Initial spacecraft drag coefficient for chaser and target (two plates deployed)

C_{max} = Maximum spacecraft drag coefficient for chaser and target (four plates deployed)

C_{min} = Minimum spacecraft drag coefficient for chaser and target (zero plates deployed)

$\boldsymbol{e}$ = Tracking error vector

e_0 = Time-varying eccentricity of the Harmonic Oscillator Motion before Rendezvous

$f(\boldsymbol{x})$ = Nonlinearities in the spacecraft dynamics

$\underline{I}_{nxn}$ = nxn Identity Matrix

i_T = Target initial spacecraft orbit inclination

J_2 = Second-order harmonic of Earth gravitational potential field (Earth flattening) [108263 × 10−8, Ref. [1]]

l = Linearized time rate of change of the amplitude of the cross-track separation (Coefficient from the Schweighart and Sedwick equations)

m_S = Spacecraft mass

n = Mean motion (Coefficient from the Schweighart and Sedwick equations)

$\underline{\boldsymbol{P}}$ = Solution matrix of the Lyapunov equation

q = Linearized argument of the cross-track separation (Coefficient from the Schweighart and Sedwick equations)

$\underline{\boldsymbol{Q}}$ = Selected Lyapunov equation matrix

$\underline{\boldsymbol{Q}}_{\boldsymbol{LQR}}$ = Matrix from the LQR problem

R = Earth mean radius (6378.1363 km, Ref. [1])

$R_{\boldsymbol{LQR}}$ = Constant from the LQR problem

r_T = Target initial spacecraft reference orbit radius

S_o = Initial spacecraft cross-wind section area for chaser and target (two plates deployed)

S_{max} = Maximum spacecraft cross-wind section area for chaser and target (four plates deployed)

S_{min} = Minimum spacecraft cross-wind section area for chaser and target (zero plates deployed)

$\boldsymbol{u}$ = Control vector

V = Lyapunov function

V_r = Spacecraft velocity vector magnitude with respect to the Earth's atmosphere

v_T = Initial anomaly of the target spacecraft orbit

u_x, u_y, u_z = Control variables expressed as relative accelerations in the LVLH orbital frame

$\boldsymbol{x}$ = State space vector including relative position and velocity of the spacecraft system in the LVLH orbital frame

x, y, z = Spacecraft system relative position in the LVLH orbital frame

$\dot{x}, \dot{y}, \dot{z}$ = Spacecraft system relative velocity in the LVLH orbital frame

$\boldsymbol{x_d}$ = Linear guidance state space vector

$\boldsymbol{z}$ = Transformed spacecraft relative position vector

$\varDelta$ = term in the derivative of the Lyapunov function

$\varDelta BC$ = Ballistic coefficient differential

$\varDelta t$ = Switching time for the differential-drag during the second phase

$\varDelta t_w$ = Waiting time between the two phases of the rendezvous maneuver

φ = Initial phasing angle for the cross-track motion

ρ = Atmospheric density

v_t = Target initial mean anomaly

Ω_T = Target initial Right Ascension of Ascending Node

ω = Target spacecraft orbital angular rate

ω_R = Angular velocity of the rotating coordinate system

ω_T = Target initial argument of perigee

I. Introduction

THIS paper presents a Lyapunov-based control strategy for the rendezvous maneuver of a chaser and a target spacecraft using aerodynamic differential drag. Control of space rendezvous maneuvers is an increasingly important topic, given the potential for its application in operations such as autonomous guidance of satellite swarms, on-orbit maintenance missions, refueling and autonomous assembly of structures in space. Several control strategies for spacecraft rendezvous maneuvers using thrusters have been developed in the past few years (see references [2] and [3]). Nevertheless, given the high cost of refueling, an alternative for thrusters as the source of the control forces is desired. A differential in the aerodynamic drag experienced by the target and chaser spacecraft produces a differential in acceleration between the spacecraft, which can be used to control the motion of the chaser relative to the motion of the target spacecraft. The concept of spacecraft maneuvering using differential drag was first proposed by C. L. Leonard [4]. The main advantage of differential drag maneuvering is that it does not require the use of any type of propellant. However, these maneuvers can only be performed at low earth orbits (LEO), where there are enough atmospheric particles to generate sufficient drag forces. An application of these principles is the JC2Sat project developed by the Canadian and Japanese Space Agencies (see references [5] and [6]).

The reference frame commonly employed for spacecraft relative motion representation is the Local Vertical Local Horizontal (LVLH) reference frame, where x points from Earth to the reference satellite (virtual or real), y points along the track (direction of motion), and z completes the right-handed frame (see Fig. 1).

To generate the drag differential, the chaser and target spacecraft must have different cross- sectional areas. A simple way of achieving this is to provide the spacecraft with a system of rotating flat plates which in practice can be solar panels. Three cases for the configurations (see Fig. 1) of the plates are considered. In the first case all the plates of the chaser are deployed generating the maximum possible drag while those of the target are not deployed to achieve the opposite, generating a negative acceleration of the chaser relative to the target. The second case is the opposite (the plates are configured so that the chaser experiences minimum drag and the target maximum), which induces a positive relative acceleration of the chase relative to the target. In the third case both chaser and target have a couple of plates deployed, which means that there is no relative acceleration between the spacecraft. The time it takes for the plates to rotate to their extreme positions (maximum and minimum drag) is very small in comparison with the time required for the orbits of the spacecraft to change significantly under the differential drag; thus, as a simplification, it is assumed that the plates rotate instantly, in other words the rotation has a bang-off-bang nature as suggested in [7], [8], and [9]. All previously cited works are based on assuming linear dynamics between the chaser and the target, neglecting the nonlinearities of the relative motion, and neglecting variability in the atmospheric density, navigation errors, and sources of disturbances on orbit.

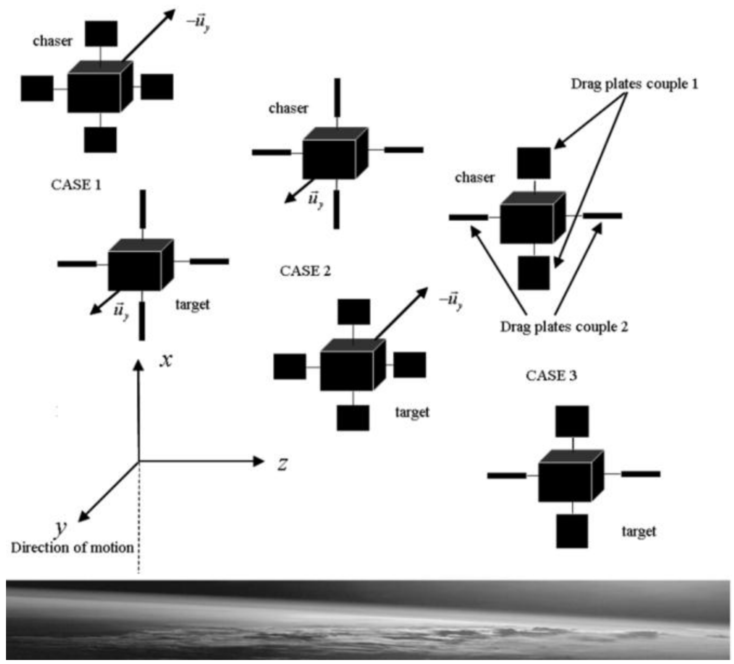


Fig. 1 Drag plates concept to generate differential drag (obtained from [9]).

In order to cope with the limitations of the above mentioned assumptions a Lyapunov approach for forcing nonlinear systems, controlled by on-off actuators only, to track trajectories is here proposed. This approach has been presented and tested in previous work by one of the authors [8].

The control strategy selects the sequence of positive, negative or zero differential accelerations of the chaser relative to the target. This selection is designed so that the dynamics of the spacecraft system (chaser and target spacecraft) tracks a given trajectory. The main criterion for the design of the control sequence is that the time derivative of a Lyapunov function of the tracking error remains negative while the dynamics of the spacecraft system tracks the linear reference model, which puts constraints on the possible relative accelerations. This significantly simplifies the control problem, since the desired trajectory (in this case the trajectory for the rendezvous maneuver) can be designed using linear control techniques on the linear model. Moreover, the use of this control method allows for the design of a smoother control action with less switching in the differential drag than previous work of one of the authors [9].

A stable linear reference model can be used to track the desired generated trajectory. The Lyapunov controller can be used to force the nonlinear model to directly track the desired generated trajectory or track the dynamics of the linear reference model. The Lyapunov controller is implemented in three following configurations:

1) The Lyapunov controller is used to force the nonlinear system to directly track the analytically generated guidance trajectory
2) The Lyapunov controller forces the nonlinear system to track the trajectory of the reference model which is regulated, thus driving both models to the origin
3) The Lyapunov controller forces the nonlinear system to track the trajectory of the reference model which is tracking the analytically generated guidance trajectory.

The foremost contributions in this work are:

1) A control strategy, based on the Lyapunov approach, for two spacecraft rendezvous using differential drag.
2) Demonstration of feasibility of the approach via numerical simulations.
3) Assessment of the performances of the designed control strategy in terms of the accuracy in the tracking of the desired trajectory and the number of switches in the differential drag caused by the control strategy.

This work is organized as follows: Section II presents the spacecraft system dynamics explaining the effect of aerodynamic drag, Section III illustrates the guidance used, Section IV comments on the Linear Reference and Nonlinear Models used, Section V explains the development of the Lyapunov based control strategy, Section VI contains the simulations performed, and finally Section VII presents the conclusions.

II. Relative Motion Spacecraft Dynamics

Hill's seminal paper on lunar theory [10] which described the motion of the moon relative to the earth was the first study on the relative motion of bodies in space. Based on Hill's work, Clohessy and Wiltshire [11] developed the linear model that bears their name, which describes the motion of a chaser spacecraft relative to a target spacecraft. This model has been widely used in applications involving low thrust proximity maneuvers. Conversely, this model does not account for the differential effects on the spacecraft motion due to nonlinearities such as the J_2 perturbation, caused by the earth's flattening. The effect of the J_2 perturbation and other nonlinearities is more significant in maneuvers with longer times of execution such as those performed using differential drag. For this reason the use of a linear model that partially accounts for averaged effects of these nonlinearities is desired, as the one described in the following section.

A. Schweighart and Sedwick Relative Motion Equations

A linearized model which represents the relative motion of spacecraft under the influence of the J_2 was developed by Schweighart and Sedwick [12]. By simply adding the control acceleration vector (**u**) to the Schweighart and Sedwick equations the following system of linear differential equations in the LVLH is obtained:

$$\ddot{x} - 2nc\dot{y} - (5c^2 - 2)n^2 x = u_x \tag{1}$$

$$\ddot{y} + 2nc\dot{x} = u_y \tag{2}$$

$$\ddot{z} + q^2 z = 2lq\cos(qt + \varphi) + u_z \tag{3}$$

Where the c and n are defined as follows:

$$c = \sqrt[2]{1 + \frac{3J_2 R}{8{r_T}^2}[1 + 3\cos(2i_t)]} \tag{4}$$

$$n = \frac{\omega_R}{c} \tag{5}$$

This set of equations is used to generate the desired trajectory for the rendezvous maneuver. It is important to note that equations (1), (2), and (3) assume circular reference orbit, a small separation between target and chaser in comparison to the radii of their orbits, and that only J_2 effects, drag and 2-body forces are acting on the spacecraft system. Moreover, the Schweighart and Sedwick equations do not provide any information related to the attitude of the spacecraft, and in this paper it is assumed that attitude is stabilized by other means than the differential drag.

B. Differential Drag

At low earth orbits (LEO) there is still a significant amount of atmospheric particles, which induces a pressure on the surface of any object moving at those orbits. In others words at LEO atmospheric density (ρ) is large enough to induce aerodynamic drag against the motion of a spacecraft. The relative acceleration caused by the differential aerodynamic drag for the spacecraft system is given in [8] as:

$$a_D = \frac{\rho \Delta BC}{2} {V_r}^2 \tag{6}$$

where the ballistic coefficient ΔBC is given by:

$$\Delta BC = \frac{C_{\max} S_{\max} - C_{\min} S_{\min}}{m_S} \tag{7}$$

Since this acceleration is caused by drag, then it only acts in the direction opposite to motion (negative y direction in the LVLH frame). This means that the only nonzero component of the control vector is u_y. Hence the control vector is given as:

$$\boldsymbol{u} = [0 \; -a_D \;\; 0]^T \tag{8}$$

It can be observed from equations (1), (2), and (3) that the dynamics in the x and y directions are independent of those on the z direction. Also from equation (8) it can be observed that control can be achieved only on the along-track direction of orbital motion (y direction). This indicates that by using differential drag, only motion in the x-y plane, with its velocities, can be controlled for the dynamics described by the Schweighart and Sedwick equations.

C. Transformation of the Schweighart and Sedwick equations

Taking into account only equations (1) and (2) the state space representation of the Schweighart and Sedwick equations is defined in equation (9). The motion of the spacecraft system outside of the x-y plane is not considered since the drag cannot influence it.

$$\dot{\boldsymbol{x}} = \underline{\mathbf{A}} + \underline{\boldsymbol{B}}\boldsymbol{u} \tag{9}$$

with matrices $\underline{\boldsymbol{A}}$, $\underline{\boldsymbol{B}}$ and vector state $\boldsymbol{x}$ defined as follows:

$$\underline{\mathbf{A}} = \begin{bmatrix} \underline{\mathbf{0}}_{2x2} & & \underline{\mathbf{I}}_{2x2} & \\ b & 0 & 0 & a \\ 0 & 0 & -a & 0 \end{bmatrix}, \;\; \underline{\boldsymbol{B}} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \;\; \boldsymbol{x} = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix} \tag{10}$$

where a, b, and d are defined as:

$$a = 2nc, \;\; b = \left(5c^2 - 2\right)n^2, \;\; d = \sqrt{a^2 - b} \tag{11}$$

Matrix $\underline{\boldsymbol{A}}$ is not Hurwitz, consequently the Schweighart and Sedwick linear model is not stable, which means that it cannot be used as a reference model for the nonlinear dynamics. The state space representation of the Schweighart and Sedwick equations can be decomposed into a double integrator and a harmonic oscillator as described in [13], resulting in a new state vector **z** using the following transformation:

$$\boldsymbol{z}=\begin{bmatrix} 0 & 1 & \frac{-a}{d^2} & 0 \\ \frac{-ab}{d^2} & 0 & 0 & \frac{-b}{d^2} \\ 0 & 0 & \frac{a^2}{2d^3} & 0 \\ \frac{-a^2b}{2d^3} & 0 & 0 & \frac{a^3}{2d^3} \end{bmatrix}\boldsymbol{x} \tag{12}$$

The solution to the state space system (12) can be found in [8] and can be expressed as:

$$z_1 = z_1|_{t=0} + t\, z_2|_{t=0} - \frac{bt^2}{2d^2}u_y \tag{13}$$

$$z_2 = z_2|_{t=0} - \frac{bt}{d^2}u_y \tag{14}$$

$$z_3 = cos(dt)\, z_3|_{t=0} + \frac{sin(dt)}{d} z_4|_{t=0} + \frac{a^3\left[1-cos(dt)\right]}{2d^5}u_y \tag{15}$$

$$z_4 = -dsin(dt)\, z_3|_{t=0} + cos(dt)\, z_4|_{t=0} + \frac{a^3\left[1-cos(dt)\right]}{2d^4}u_y \tag{16}$$

III. Guidance

An analytical guidance for the rendezvous problem can be found by linearizing the dynamics of the spacecraft system resulting in the Schweighart and Sedwick equations. This results in a linear model for the system. Afterwards, a trajectory that results in the states of the Schweighart and Sedwick model reaching the origin in the state space is designed (zero differential position and velocity between the spacecraft).

A. Trajectory for the Schweighart and Sedwick model

The rendezvous maneuver can be preliminarily separated into two sequential phases, as previously suggested in the literature ([8] and [9]). In the first phase the spacecraft are driven towards a stable relative orbit; in the second phase the oscillation of the relative orbit is canceled out and the rendezvous conditions are achieved, namely zero relative position and velocity. In order to find this solution, the state vector transformation from the LVLH coordinate to the new set of coordinates $\boldsymbol{z} = [z_1\ z_2\ z_3\ z_4]^T$ appears to be convenient since it provides a representation of the system behavior in which the dynamics of the system are decoupled into a double integrator and a harmonic oscillator. The transformation is linear and it does not shift the origin, which means that to satisfy the rendezvous condition the transformed coordinates (***z***) must reach zero for rendezvous to occur.

In the transformed coordinate system the first phase of the maneuver drives the z_1 and z_2 components to zero. This process is demonstrated in Fig. 2 a) in which starting from state α, a differential acceleration between the vehicles is induced. Due to its bang-off-bang nature this differential acceleration can only be negative, positive or zero. The sign of this relative acceleration is changed at states β, γ, and δ thus driving z_1 and z_2 to the origin in the z_1-z_2 plane.

The second phase consists of driving z_3 and z_4 to zero. This results in z_1 and z_2 deviating from the origin in the z_1-z_2 plane; however, by applying the negative and positive relative accelerations, required to drive z_3 and z_4 to the origin in the z_3-z_4 plane, for exactly the same time intervals, the z_1 and z_2 coordinates will reach the origin in the z_1-z_2 plane at the end of the second phase. The second phase, only for the z_3-z_4 plane, is illustrated in Fig. 2 b) in which, starting from state ε no relative acceleration is applied until state ς reached. This inactive period of Δt_w is desired in order to assure that the system follows a trajectory in which the time interval for positive and negative relative accelerations is the same. As shown in [9] the value of Δt_w can be found as follows:

$$\Delta t_w = \frac{1}{d}\left| \tan^{-1}\left(\left.\frac{z_3}{z_4}\right|_{\varsigma} \right) - \tan^{-1}\left(\left.\frac{z_3}{z_4}\right|_{\varepsilon} \right) \right| \tag{17}$$

Afterwards, starting from state ς a relative acceleration is again induced. Then at states η and θ the sign of the relative accelerations is changed after time intervals of Δt and $2\Delta t$ respectively. As shown by [8] this time interval Δt can be calculated using:

$$\Delta t = \frac{1}{d}\cos^{-1}\left[\frac{1}{2}\left(1 + \sqrt{h} - \sqrt{3 - h - \frac{2}{\sqrt{h}}} \right) \right] \tag{18}$$

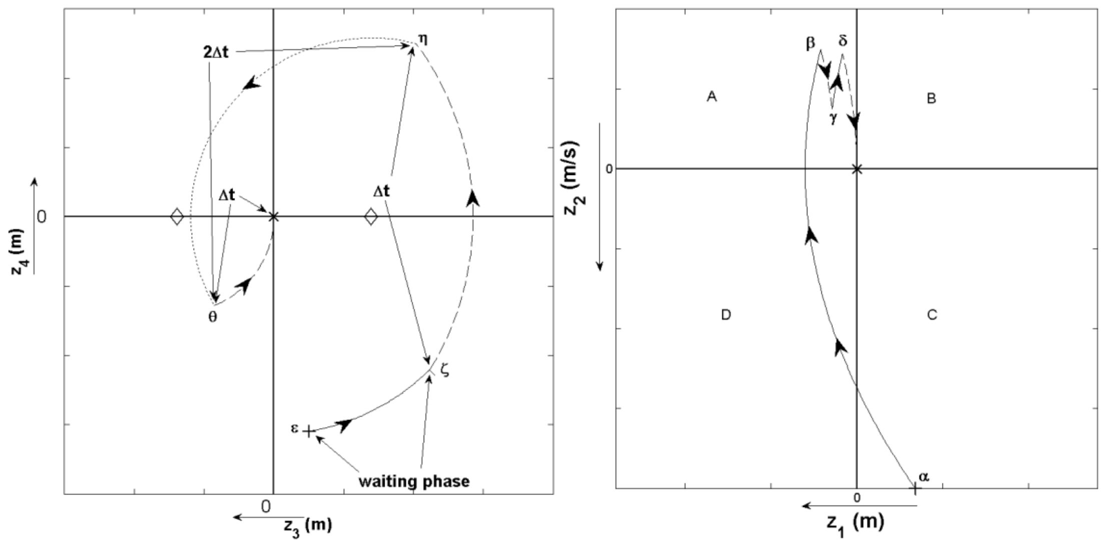
where coefficients h, f, and g are given as:

$$h = 1 + \frac{\sqrt[3]{f}}{6g} - \frac{e_0}{g\sqrt[3]{f}} \tag{19}$$

$$f = -54ge_0^{\ 2} + 6\sqrt{3}e_0^{\ 2}\sqrt{2e_0^{\ 2} + 27g^2} \tag{20}$$

$$g = -\frac{\sqrt{2}a^3 \left| u_y \right|}{2d^5} i \tag{21}$$

Subsequently, the sign of the relative acceleration is maintained for another Δt time interval after which the origin in the z_3-z_4 plane is reached. Since positive and negative relative accelerations are maintained for net time intervals of 2Δt, the origin is reached in both z_1-z_2 and z_3-z_4 planes. Once the trajectory for the two phases in the transformed coordinates is found, the analytical control sequence is translated back into the original x, y reference, and the guidance trajectory is generated.



a) First phase: stabilization of the relative orbit, **b) Second phase: final rendezvous, canceling out residual oscillations.**

Fig. 2 Two-phase differential drag guidance (obtained from [8]):

Furthermore the time-varying eccentricity of the harmonic oscillator motion before rendezvous is given as:

$$e_0 = \sqrt{z_3^{\ 2} + \left(\frac{z_4}{d}\right)^2} \tag{22}$$

As indicated in [8] at the end of the first phase e_0 must be smaller than e_c otherwise the inverse cosine in equation (18) cannot be evaluated. This critical value was determined in [8] to be:

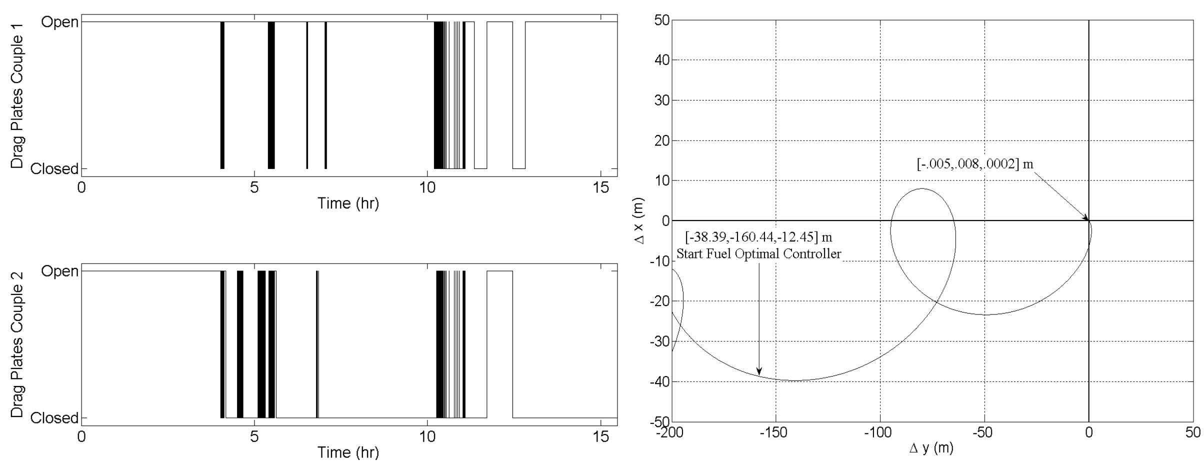
$$e_c = \frac{13a^3 \left|u_y\right|}{5d^5} \tag{23}$$

As recommended in [8] if at the end of the first phase e_0 is larger than e_c then e_0 is corrected using the following equation:

$$e_0 = e_0 - 0.99e_c \tag{24}$$

And then equation (18) is used and phase two is performed as illustrated before. The second phase is repeated until e_0 is larger than e_c and then the final second phase is executed driving all the **z** states to zero. It is important to note that in a real-world application of this guidance the tracking error is not expected to reach zero, but to reach a residual value near the origin. The reason for this behavior is the on-off nature of differential drag which does not allow for a smooth control action.

The real world problem of designing a control system for the rendezvous maneuver using differential drag becomes the problem of designing a feedback control law for the flat panels, forcing the satellites to follow the two-phase guidance (see Fig. 2) coping with nonlinearities, uncertainties, and navigation errors. Previous results [8] suggest control implementations to track the guidance shown in Fig. 2 that results in satisfactory differential drag based rendezvous maneuvers. However, some thrusting capability is required to complete the mission, due to small inaccuracies in guidance tracking, and a high number of switching commands to the drag plates (see Fig. 3). These limitations translate into a non-propellant-free maneuver, and demanding energy requirements for the plates' actuators.



a) Drag plates command sequence (see also Fig. 1)

b) Relative position in the x-y plane showing the residual distance between the satellites after differential drag control, requiring for a final thrusters powered maneuver.

Fig. 3 Previous results from reference ([8]):

IV. Linear Reference and Nonlinear Models

A. Linear Reference Model

The Schweighart and Sedwick model (as defined in equation (9)) is used to create the stable reference model. Since the dynamics of the Schweighart and Sedwick model is unstable, a Linear Quadratic Regulator (LQR) feedback controller is used to stabilize them. The resulting reference model is described by:

$$\dot{\boldsymbol{x}}_d = \underline{\boldsymbol{A}}_d \boldsymbol{x}_d, \quad \underline{\boldsymbol{A}}_d = \underline{\boldsymbol{A}} - \underline{\boldsymbol{B}}\underline{\boldsymbol{K}}, \quad \boldsymbol{x}_d = \begin{bmatrix} x_d & y_d & \dot{x}_d & \dot{y}_d \end{bmatrix}^T \tag{25}$$

Where $\underline{\boldsymbol{A}}$ and $\underline{\boldsymbol{B}}$ are defined in equation (10), and $\underline{\boldsymbol{K}}$ is a constant matrix found by solving the LQR problem for the Schweighart and Sedwick model, thus ensuring $\underline{\boldsymbol{A}}_{\boldsymbol{d}}$ to be Hurwitz. The $\boldsymbol{Q_{LQR}}$ matrix and $R_{\boldsymbol{LQR}}$ value used to solve the LQR problem are:

$$\underline{\boldsymbol{Q}}_{LQR} = \underline{\mathbf{I}}_{\boldsymbol{4x4}}, \quad R_{LQR} = 10^{19} \tag{26}$$

It is worth mentioning that the state vector $\boldsymbol{x_d}$ is the desired reference trajectory, and control action is along the y direction only. This stable linear reference system can be regulated or forced to track the guidance described before.

B. Nonlinear Model

The dynamics of spacecraft relative motion is nonlinear due to effects such as the J_2 perturbation, and the nonlinear variations on the atmospheric density at LEO. Navigation errors also contribute in increasing uncertainty with respect to the analytical linear solution previously presented. The Lyapunov based approach here suggested intends to cope with these limitations. In this section the preliminary model that will be used for the nonlinear dynamics and for testing the control strategy is presented. The general expression for the real world nonlinear dynamics, including nonlinearities such as the J_2 perturbation, is defined as:

$$\dot{\boldsymbol{x}} = \boldsymbol{f}\left(\boldsymbol{x}\right) + \underline{\boldsymbol{B}}\boldsymbol{u}, \ \boldsymbol{x} = \begin{bmatrix} x & y & \dot{x} & \dot{y} \end{bmatrix}^T, \qquad u = \begin{cases} a_D \\ 0 \\ -a_D \end{cases} \tag{27}$$

where a_D acts along the y direction only as explained earlier. All the nonlinearities are accounted for in the nonlinear function f(**x**).

The representation of the acceleration of a spacecraft in the inertial frame is defined as:

$$\boldsymbol{a} = \frac{-\mu \boldsymbol{r}}{r^3} + \boldsymbol{u} + \boldsymbol{a}_{J2} \tag{28}$$

where **u** is the control acceleration caused by aerodynamic drag in the inertial frame, and $\mathbf{a}_{J2}$ is the acceleration caused by the J_2 perturbation defined in the inertial frame as:

$$\boldsymbol{a}_{J2} = -\frac{3}{2}\frac{J_2 R_e^{\,2}}{r^4}\begin{bmatrix} (1-3\frac{z^2}{r^2})\frac{x}{r} & (1-3\frac{z^2}{r^2})\frac{y}{r} & (3-3\frac{z^2}{r^2})\frac{z}{r} \end{bmatrix}^T \tag{29}$$

V. Lyapunov Approach

In this section the nonlinear feedback control law based on the Lyapunov approach is designed, with the aim of mitigating the current limitations of differential drag control. The approach is inspired by previous work from one of the authors [2].

The Lyapunov function is defined as:

$$V = \boldsymbol{e}^{\boldsymbol{T}} \underline{\boldsymbol{P}} \boldsymbol{e}, \qquad \boldsymbol{e} = \boldsymbol{x} - \boldsymbol{x}_d, \qquad \underline{\boldsymbol{P}} \succ 0 \tag{30}$$

where $\underline{\boldsymbol{P}}$ is a symmetric positive definite matrix and $\boldsymbol{e}$ is the tracking error vector. The time derivative of the Lyapunov function can be manipulated, using some algebra to obtain:

$$\dot{V} = \boldsymbol{e}^T (\underline{\boldsymbol{A}}_{\boldsymbol{d}}{}^T \underline{\boldsymbol{P}} + \underline{\boldsymbol{P}}\underline{\boldsymbol{A}}_{\boldsymbol{d}})\boldsymbol{e} + 2\boldsymbol{e}^T \underline{\boldsymbol{P}}(\boldsymbol{f}(\boldsymbol{x}) - \underline{\boldsymbol{A}}_{\boldsymbol{d}}\boldsymbol{x} + \underline{\boldsymbol{B}}\boldsymbol{u} + -\underline{\boldsymbol{B}}\boldsymbol{u}_{\boldsymbol{d}}) \tag{31}$$

If matrix $\underline{\boldsymbol{A}}_{\boldsymbol{d}}$ is Hurwitz, a symmetric positive definite matrix $\underline{\boldsymbol{Q}}$ can be found such that it satisfies Lyapunov equation (32). This is the reason why the reference model must be stable.

$$-\underline{\boldsymbol{Q}} = \underline{\boldsymbol{A}}_{\boldsymbol{d}}{}^T \underline{\boldsymbol{P}} + \underline{\boldsymbol{P}}\underline{\boldsymbol{A}}_{\boldsymbol{d}} \tag{32}$$

Choosing a $\underline{\boldsymbol{Q}}$ matrix satisfying equation (32), results in the following expression:

$$\dot{V} = -\boldsymbol{e}^{\boldsymbol{T}} \underline{\boldsymbol{Q}} \boldsymbol{e} + 2\Delta \tag{33}$$

where Δ is given by:

$$\Delta = \beta \hat{u} - \delta \tag{34}$$

and β, δ and û (the command sent to the plate actuators) are given by the following expressions:

$$\beta = \boldsymbol{e}^{\boldsymbol{T}} \underline{\boldsymbol{P}}\underline{\boldsymbol{B}} a_D, \quad \delta = \boldsymbol{e}^{\boldsymbol{T}} \underline{\boldsymbol{P}}\left(\underline{\boldsymbol{A}}_{\boldsymbol{d}}\boldsymbol{x} - \boldsymbol{f}(\boldsymbol{x}) + \underline{\boldsymbol{B}}\boldsymbol{u}_{\boldsymbol{d}}\right), \quad \hat{u} = \begin{cases} 1 \\ 0 \\ -1 \end{cases} \tag{35}$$

A. Control Strategy

Guaranteeing $\Delta < 0$ would imply that the tracking error (**e**) converges to zero, since the term involving $\underline{\boldsymbol{Q}}$ is already negative. In other words, as long as $\Delta < 0$, the system dynamics of the spacecraft will track the desired trajectory. However, since the control variable û is only present in β, the system cannot be guaranteed to be Lyapunov stable for the chosen Lyapunov function if δ is positive, and has a higher magnitude than β. The magnitude of β is linearly dependent on the atmospheric density ρ, which indicates that if ρ is too small (higher orbits) the system is unstable since β will not have a magnitude large enough to overcome a positive value of δ of great magnitude. In others words, the motion of the spacecraft cannot be controlled if ρ is not large enough.

The activation strategy for the control is designed to such that the chosen value of û forces β to be negative, thus û can be expressed as:

$$\hat{u} = -sign(\beta) \tag{36}$$

It is worth emphasizing that all the components in the above activation strategy would be available in real time on board a spacecraft. A diagram of the proposed control scheme is shown in Fig. 4.

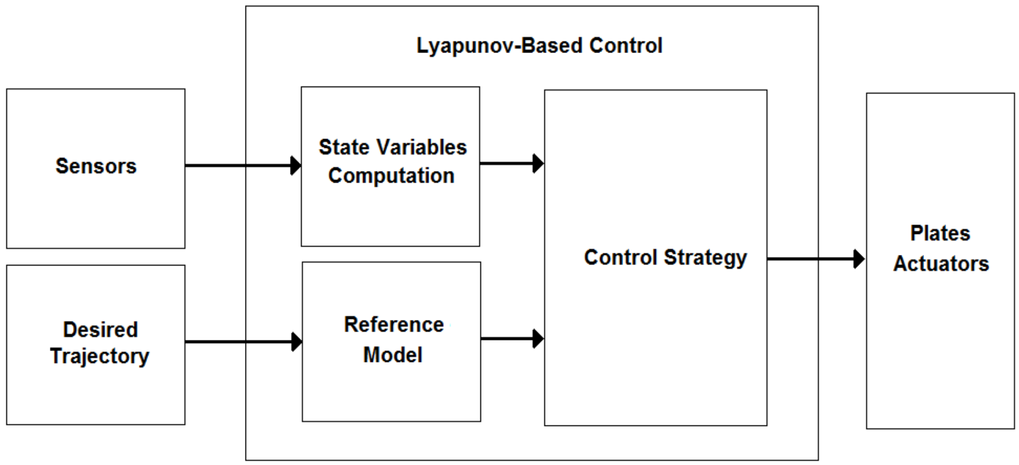


Fig. 4 Control diagram

VI. Numerical Simulations

The proposed technique is intended to cope with nonlinearities, such as the J_2 perturbation, and uncertainties in the atmospheric density, and to show resilience against relative navigation errors. This is due to its feedback nature, and the inclusion of the tracking error in the activation logic. The initial orbital elements of the target and other parameters for the numerical simulations are shown in Table 1. The target and chaser spacecraft are assumed to be identical, therefore drag coefficient and cross-wind section areas for all plate configurations are the same. The initial relative position and velocity of the chaser in the LVLH frame are shown in Table 2. The simulations validate the approach taking into account variable atmospheric density, and J_2 perturbations, to represent the nonlinear behavior of the spacecraft system.

Table 1 Simulation Parameters and Target Initial Orbital Elements

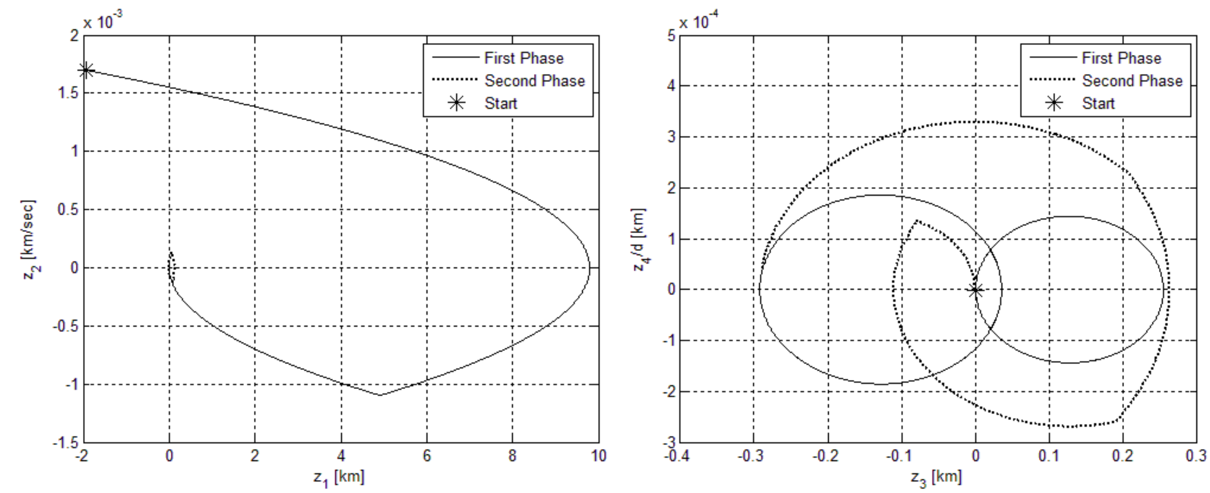
Parameter	Value
J_2	108263E-08
R (km)	6378.1363
i_T (deg)	97.99
Ω_T (deg)	261.62
ω_T (deg)	30
ν_T (deg)	25.02
r_T (km)	6778.1
μ(km^3/sec^2)	398600.4418
m_S (kg)	10
S_{max} (m^2)	2.1
S_{min} (m^2)	0.5
S_o (m^2)	1.3
C_{Dmax}	2.5
C_{Dmin}	1.5
C_{Do}	1

Table 2 Chaser initial position, and velocity in the LVLH

Parameter	Value
x (km)	-1
y (km)	-2
$\dot{x}$ (cm/sec)	0.0483
$\dot{y}$ (m/sec)	169.7126

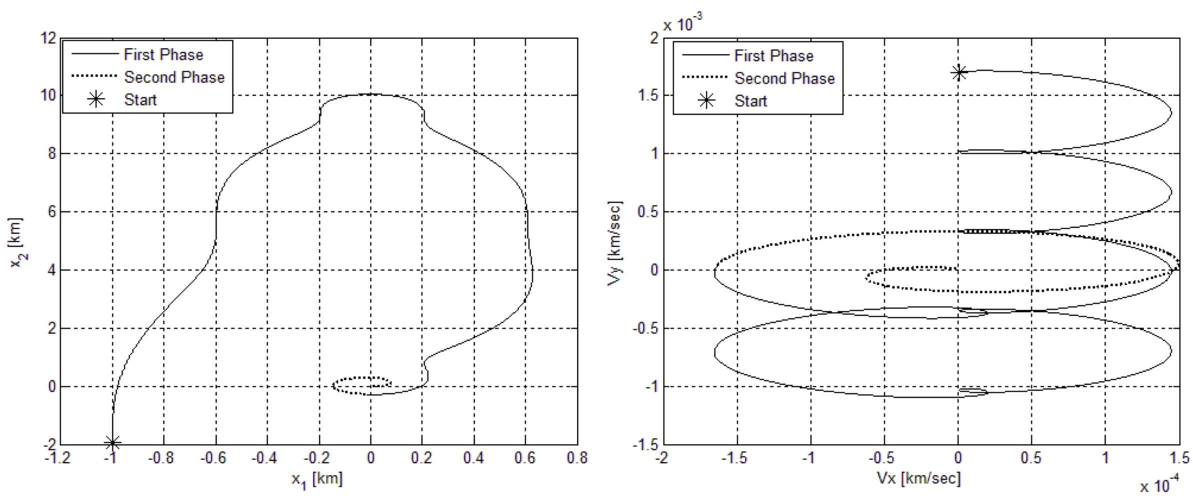
For the simulations, a MATLAB program that analytically generates the guidance trajectories and their respective control sequences was created. The nonlinear dynamics of the system in the inertial frame was propagated using Simulink models. Afterwards, all state variables are transformed to the LVLH frame and the controls strategy is implemented. The atmospheric model used to generate ρ as function of the latitude, longitude and altitude of the spacecraft is the NRLMSISE-00 found in the Simulink Aerospace Block set. To better represent a real maneuver, the Lyapunov controller was activated every 10 minutes and a waiting period of 15 minutes between changes in the plate deployment was set. Moreover, the rendezvous maneuver was assumed to be finalized when the inter spacecraft distance was below 10m.

A two phase rendezvous trajectory, as described in section III, was generated using the parameters and initial conditions in Table 1, and Table 2, and the Schweighart, and Sedwick model. This trajectory in the **z** and **x** coordinate systems can be seen in Fig. 5 and Fig. 6 respectively.



a) z_1-z_2 plane **b) z_3-z_4 plane**

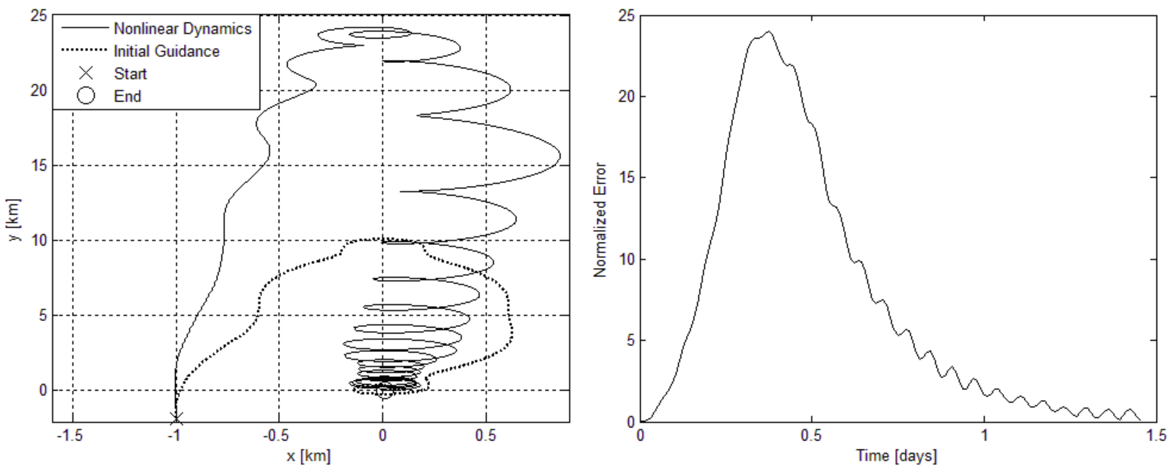
Fig. 5 Generated Trajectory in the z coordinates



a) x-y plane **b) $\dot{x}-\dot{y}$ plane**

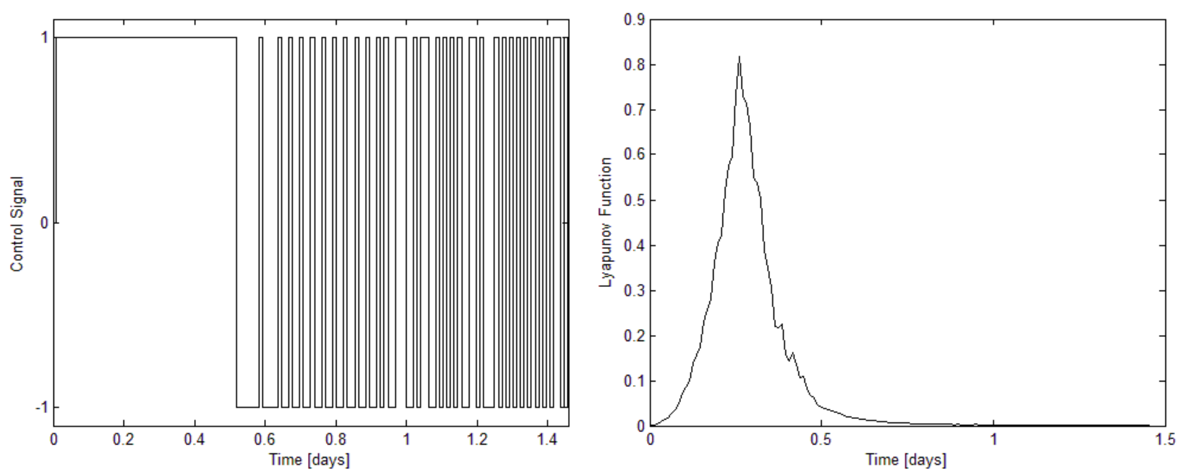
Fig. 6 Generated Trajectory in the x coordinates

Three main configurations for the simulations were used. In the first configuration the Lyapunov controller was used to force the nonlinear system to directly track the guidance trajectory. The trajectory in x-y plane, the normalized tracking error, the control sequence and the value of the Lyapunov function for this configuration are shown in Fig. 7 and Fig. 8.



a) Trajectories x-y plane, **b) Normalized Tracking Error**

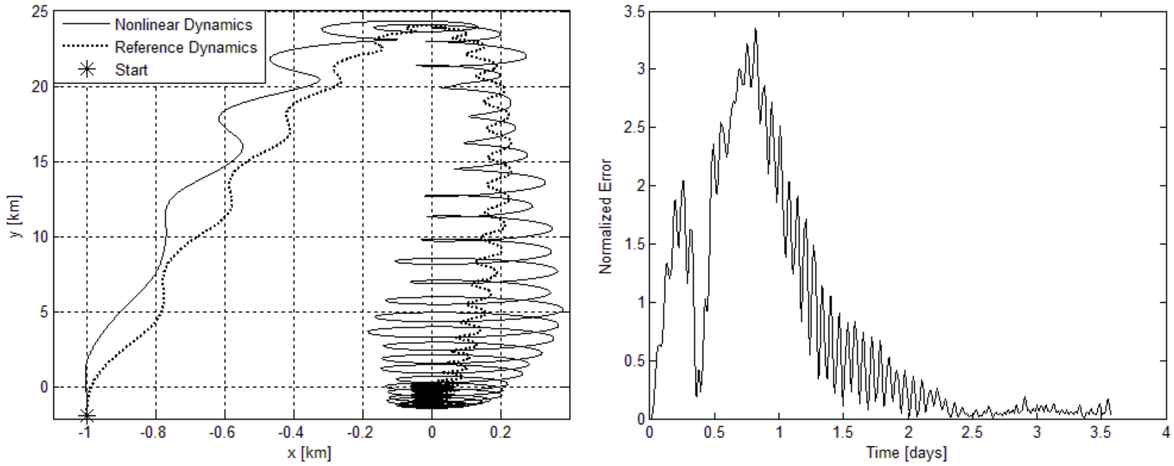
Fig. 7 First simulation configuration



a) Control Sequence **b) Lyapunov function**

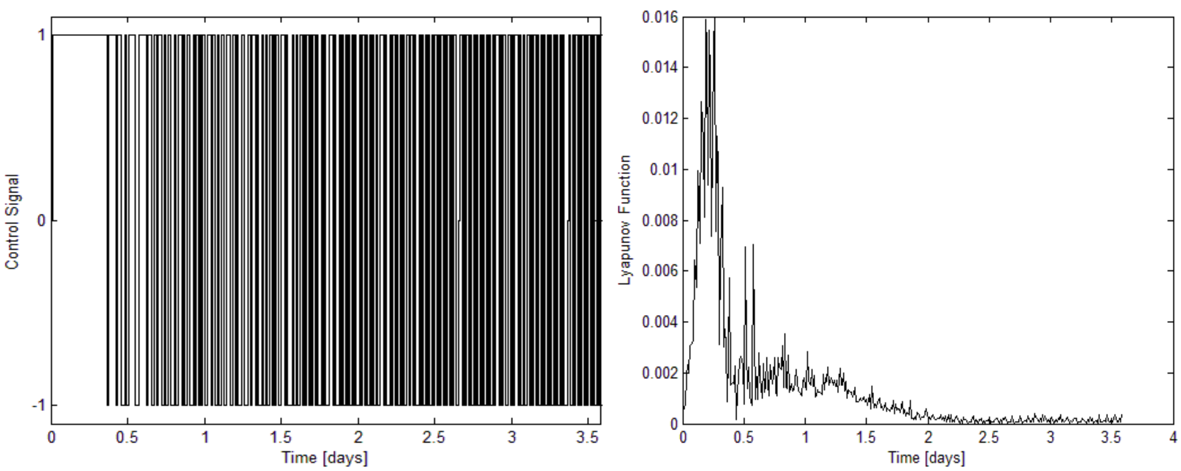
Fig. 8 First simulation configuration

In the second configuration the Lyapunov controller forces the nonlinear system to track the trajectory of the reference model which is regulated, thus driving both models to the origin (rendezvous state). In this configuration the guidance trajectory is not used. The trajectory in x-y plane, the normalized tracking error, the control sequence and the value of the Lyapunov function for this configuration are shown Fig. 9 and Fig. 10.



a) Trajectories x-y plane, **b) Normalized Tracking Error**

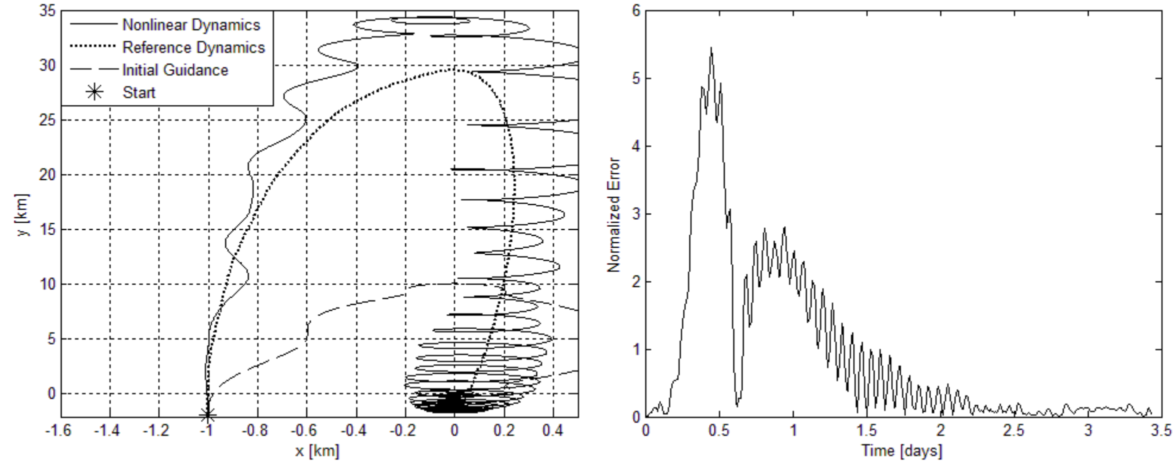
Fig. 9 Second simulation configuration



a) Control Sequence **b) Lyapunov function**

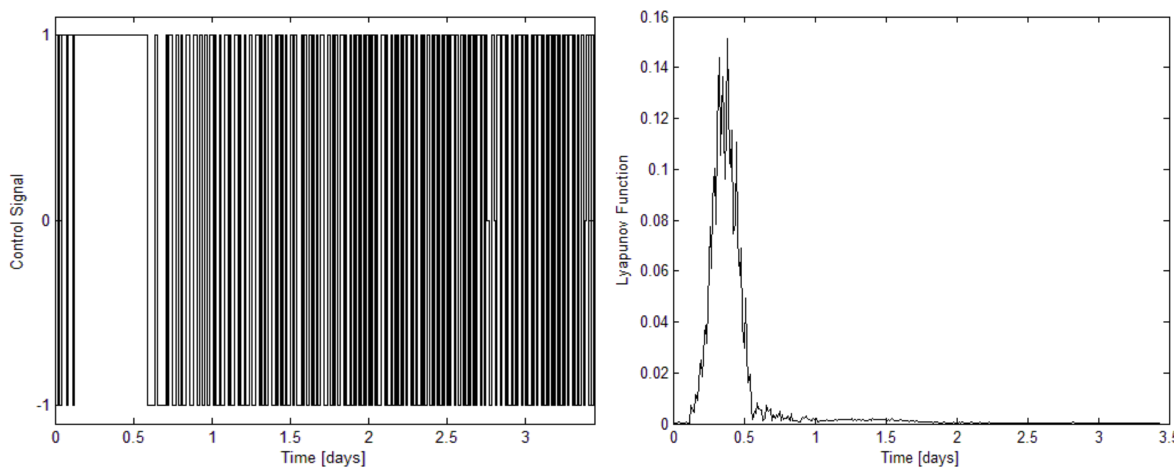
Fig. 10 Second simulation configuration

In the third configuration the Lyapunov controller forces the nonlinear system to track the trajectory of the reference model which is tracking the guidance trajectory. The trajectory in x-y plane, the normalized tracking error and the control sequence and the value of the Lyapunov function for this configuration are shown in Fig. 11 and Fig. 12.



a) Trajectories x-y plane, b) Normalized Tracking Error

Fig. 11 Third simulation configuration



a) Control Sequence b) Lyapunov function

Fig. 12 Third simulation configuration

A summary of main results for the simulations is presented in Table 3. In all the simulations the Lyapunov controller was able to force the system to accurately track the desired trajectory, driving the spacecraft to within 10 meters of each other. For all three configurations the Lyapunov function presented an increasing tendency at the beginning of the maneuver (first 12 hrs), which indicates that in that interval, the time derivative of the Lyapunov function was positive. Afterwards, the Lyapunov function exhibits an overall decaying tendency up until the end of the maneuver with brief intervals of increasing tendency. This indicates that during some intervals of the maneuver (especially during the first 12 hr) Lyapunov stability cannot be proved using the proposed Lyapunov function. The best results were obtained by using the Lyapunov controller to track directly the guidance trajectory (First

Configuration), which resulted in maneuver with only 64 changes in the control and was performed in less than a day.

Table 3 Summary of Simulations Results

Simulation	Number of Changes in Control	Maneuver Duration (hr)
First Configuration	64	35
Second Configuration	251	86
Third Configuration	218	83

VII. Conclusion

In this work a new application of the of Lyapunov principles for the autonomous control of a spacecraft rendezvous maneuver by making use of differential drag is presented. By varying the differential drag between the chaser, and target satellites their motion in the x-y and V_x-V_y plane in the LVLH frame is controlled. These variations are induced by the action of sets of plates fixed to the spacecraft. The nature of this variation is assumed to be of bang-off-bang nature with only three possible values: maximum differential acceleration, minimum differential acceleration, and zero differential acceleration. Consequently, the control action is a sequence of these three values. A trajectory of the rendezvous maneuver is generated using the Schweighart and Sedwick model.

Lyapunov principles are used to develop criteria for the activation of the actuators of the plates which generate the differential drag. These criteria are designed so that the Lyapunov function of the tracking error is positive, and the derivative of the Lyapunov function is negative, thus ensuring that the tracking error converges to zero.

The control method is then tested using Simulink Models for the real nonlinear dynamics. The performance of the control method is assessed in terms of the tracking accuracy, and the number of switches in the differential acceleration in the simulations. The use of a linear stable model of the system as reference model is evaluated and found to present less desirable results than tracking directly the guidance trajectory.

The resulting controlled actuation is an improvement over previous results since it presents a significantly lower frequency of actuation, while accurately tracking the desired trajectory and a reasonably small residual error.

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