# A Novel Self-localization Protocol for Spacecraft Clusters 

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## Introduction

## Multimensional Scaling

3) Find the eigenvalues and eigenvectors of $B C$

$$
B_{c}=O^{T} \Lambda O
$$

4) The 3D positions of all the satellites are finally given by the first three non-zero columns of the matrix

$$
P=O \sqrt{\Lambda}
$$

## The Algorithm

1) Satellite number 1 broadcasts the time $t 1$ of its internal clock.
2) All other satellites record the time tl and the arrival time $\tau_{1 \mathrm{x}}$ of the broadcast.

- The difference $\Delta_{\Delta_{1}=c\left(\tau_{1}-t_{1}\right)}$ gives the distance of the $x$-satellite from satellite 1

3) Satellite number 2 broadcasts the time t2 at which it receives the broadcast from satellite 1 . arrival time $\tau_{2 x}$ of the broadcast.

- The difference ${ }_{\Delta_{2}=c\left(\tau_{2}-t\right)}$ gives the distance of the x -satellite from satellite 2 .

5) At this point all satellites can also compute the distance between satellites 1 and 2 :
6) Steps 3), 4), 5) are repeated for aif remaining satellites, in turns.
7) At this point in time, a second iteration of broadcasts begin in which each $\mathrm{s} / \mathrm{c}$ broadcasts the distance between itself and every other $\mathrm{s} / \mathrm{c}$ in the cloud.
8) From the distance matrix $D$ each satellite can reconstruct the three-dimensional positions of all satellites in the cluster, by using multidimensional scaling.

## Advantages of the Algorithm

1) Given the distance matrix $D$, compute the auxiliary matrix:

$$
B_{i j}=\frac{1}{2}\left(D_{i 1}^{2}+D_{j 1}^{2}-D_{i j}^{2}\right)
$$

2) Compute the centered matrix:
$B_{c}=J B J, \quad$ where $\quad J=-\frac{1}{N}\left(\begin{array}{ccccc}1-N & 1 & 1 & 1 \\ 1 & 1-N & 1 & 1 \\ 1 & 1 & 1-N & 1 & \cdots \\ 1 & 1 & 1 & 1-N\end{array}\right)$
only
only two iterations. result of 1000 simulated noisy cycles for systems of 7 and $35 \mathrm{~s} / \mathrm{c}$ along with the positional uncertainty plotted as a function of the number of $\mathrm{s} / \mathrm{c}$.

Such an algorithm minimizes both communication and computation costs, and therefore is expected to be energy efficient. Determination of iteration cycles in the selflocalization process in critical in order to achieve an acceptable level of accuracy.

## Multidimensional Scaling

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Atmospheric Differential Drag



Spacecraft positioning within the cloud can be addressed, for example, within the context of atmospheric differential drag techniques. This technology enables reactionless formation control, guaranteeing that the entire system will have line of site communications with every other spacecraft in the cluster. The overarching cluster architecture goal is such that a single spacecraft will aggregate all data and be the source of all communications to the ground. Thus, from a ground operational perspective, communications with the system can be achieved using approximately the same resources normally allocated to communicating with individual spacecraft.

See http://www.riccardobevilacqua.com/
Atmospheric Differential Drag Control Spacecraft Formations w/o Propellant


