EAS 4510
Astrodynamics
Spring 2017

Exam 1
8 February 2017

What is Allowed During Examinations and Homework

You may use any books, your personal notes, or electronic aid, provided that you find the material on your own without having it provided to you by anyone else (either implicitly or explicitly). **You may not, under any circumstances, communicate with anyone about this assignment, and that includes me and the TAs!** Any violations of these rules will result in further action on my part in a manner consistent with the academic honesty policy of the University of Florida. The academic honesty policy can be found at the Student Conduct and Conflict Resolution website:
https://www.dso.ufl.edu/sccr/process/student-conduct-honor-code/

Guidelines for Solutions

Communication is an extremely important part of demonstrating that you understand the material. To this end, the following guidelines are in effect for all problems on the examination/homework:

1. Your handwriting must be neat. I will not try to decipher sloppy handwriting and will assume that something is incorrect if I am unable to read your handwriting.
2. **ONLY FOR IN-CLASS EXAMS:** Your exam must be HANDWRITTEN, no software, no scans, etc., your own handwriting ONLY. If anything else appears other than your own handwriting, the exam will be evaluated at 0 (zero).
3. You must be crystal clear with every step of your solution. in other words, any step in a derivation or statement you write must be unambiguous (i.e., there must be one and only one meaning). IF it is ambiguous as to what you mean in a step, then I will assume the step is incorrect.
4. Exams without a name on each page, and/or without a UFID and signature at the bottom of this page, will not be graded (i.e., they will receive a score of 0 (zero)).
5. ANY assignment (HW, exam, etc.) without signature, date, and UFID at the bottom of this page, will not be graded (i.e., they will receive a score of 0 (zero)).

In short, please write your solutions in an orderly fashion so that somebody else can make sense of what you are doing and saying. Finally, credit will be given only if a relevant concept is applied properly, and no credit will be given for an incorrectly applied concept even if the final answer is correct.

University of Florida Honor Code

On your examination/homework you must state and sign the University of Florida honor pledge as follows:

On my honor, I have neither given nor received unauthorized aid in doing this examination/homework.

Signature: **Date:**

University of Florida ID:

Exam 1

February 8, 2017

1. Using the following expression for orbital velocity,

$$v = \sqrt{\frac{\mu}{r}\left(2 - \frac{1-e^2}{1+e\cos\theta}\right)}$$

   determine the velocity of a spacecraft at

   a) perigee, $r_p = a(1-e)$
   b) apogee, $r_a = a(1+e)$

   Express your final answers in terms of a, e, and μ. **(20 Points)**

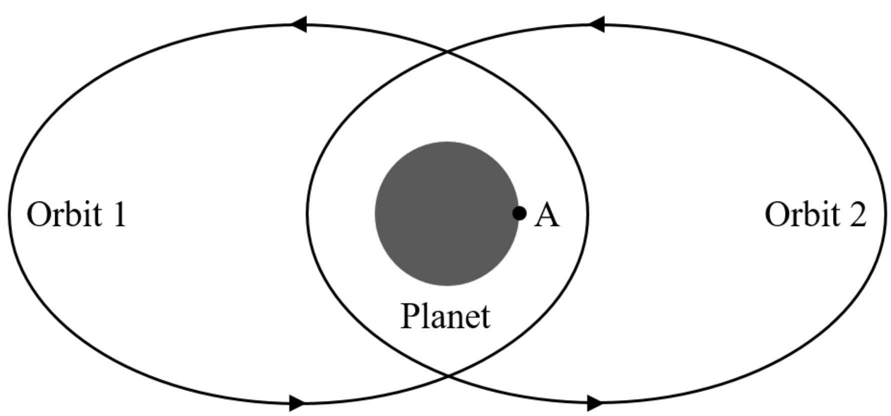
2. Spacecraft A is in a circular, equatorial orbit around the Earth with velocity $v = 7\ \frac{\text{km}}{\text{s}}$. Spacecraft B is in a separate Earth orbit with a perigee velocity of $v_p = 8\ \frac{\text{km}}{\text{s}}$. What is the eccentricity of spacecraft B's orbit if both spacecraft have the same semi-major axis, a? **(15 Points)**

3. Using state-space representation, the state of a spacecraft can be expressed as $\mathbf{s} \triangleq \begin{bmatrix} \mathbf{r} \\ \mathbf{v} \end{bmatrix}$. Determine $\mathbf{s}$ and $\dot{\mathbf{s}}$ using the following vectors:

$$\boldsymbol{r} = 10,000\hat{\boldsymbol{x}} \quad \text{km} \qquad \dot{\boldsymbol{r}} = 10\hat{\boldsymbol{y}} \quad \frac{\text{km}}{\text{s}} \qquad \ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\boldsymbol{r} \quad \frac{\text{km}}{s^2}$$

   Note: Your final answers should be 6×1 column matrices. **(15 Points)**

4. A spacecraft is tasked with photographing point A on a planet's surface. Two orbits are proposed with identical size and shape ($a_1 = a_2$, $e_1 = e_2$, $T_1 = T_2$). Assume the planet is not rotating (A is stationary with respect to both orbits). Which orbit allows the spacecraft to take images of A for a longer duration? Briefly justify your answer. **(10 Points)**



5. The International Space Station (ISS) is on a circular orbit 422 km above the Earth. A GPS satellite is on a circular orbit 20,200 km above the Earth. Which spacecraft has a greater specific energy? Briefly justify your answer. **(10 Points)**

6. Consider the two spacecraft from Problem 5. Which spacecraft has a greater orbital period? Briefly justify your answer. **(10 Points)**

7. Earth's orbit around the Sun has an average radius of about 1 AU (150 million km). Mars orbits the Sun at a distance of about 1.52 AU on average. Due to Kepler's second law ("law of areas") the orbital velocity of Mars must be greater than that of Earth's. **(True/False) (5 Points)**

8. The eccentricity vector is always perpendicular to the specific angular momentum vector. **(True/False) (5 Points)**

9. Due to the conservation of angular momentum, the product of position and velocity vector magnitudes is constant for all points on a Keplerian orbit. **(True/False) (5 Points)**

10. For elliptical orbits, the velocity vector is always perpendicular to the position vector. **(True/False) (5 Points)**

Solutions

1. Perigee:

   $r_p = a(1-e)$, $\theta = 0$

   $v_p = \sqrt{\frac{\mu}{r_p}\left(2 - \frac{1-e^2}{1+e}\right)} = \sqrt{\frac{\mu}{r_p}\left(2-(1-e)\right)} = \sqrt{\frac{\mu}{r_p}\left(1+e\right)} = \sqrt{\frac{\mu}{a}\frac{(1+e)}{(1-e)}}$

   Apogee:

   $r_a = a(1+e)$, $\theta = \pi$

   $v_a = \sqrt{\frac{\mu}{r_a}\left(2 - \frac{1-e^2}{1-e}\right)} = \sqrt{\frac{\mu}{r_a}\left(2-(1+e)\right)} = \sqrt{\frac{\mu}{r_a}\left(1-e\right)} = \sqrt{\frac{\mu}{a}\frac{(1-e)}{(1+e)}}$

2. $v_{\text{circular}} = \sqrt{\frac{\mu}{a}} \quad \rightarrow \quad a = \frac{\mu}{v^2_{\text{circular}}} \quad \rightarrow \quad a = 8134 \quad \text{km}$

   $v^2 = \mu\left(\frac{2}{r} - \frac{1}{a}\right) \quad \rightarrow \quad r = \frac{2}{\frac{v^2}{\mu}+\frac{1}{a}} \quad \rightarrow \quad r = r_p \quad \text{and} \quad v = v_p$

   $a\,(1-e) = \frac{a}{\frac{v^2}{\mu}+\frac{1}{a}} \quad \rightarrow \quad e = 1 - \frac{\frac{2}{a}}{\frac{v^2}{\mu}+\frac{1}{a}} \quad \rightarrow \quad e = 0.1327$

3. $\mathbf{s} = \begin{bmatrix}\mathbf{r}\\ \mathbf{v}\end{bmatrix} \qquad \dot{\mathbf{s}} = \begin{bmatrix}\dot{\mathbf{r}}\\ \dot{\mathbf{v}}\end{bmatrix} = \begin{bmatrix}\mathbf{v}\\ \mathbf{a}\end{bmatrix}.$

   Using $\quad \boldsymbol{r} = 10,000\hat{\boldsymbol{x}} \quad \text{km} \qquad \dot{\boldsymbol{r}} = 10\hat{\boldsymbol{y}} \quad \frac{\text{km}}{\text{s}} \qquad \ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\boldsymbol{r} \quad \frac{\text{km}}{s^2}$

   $\ddot{\boldsymbol{r}} = -\frac{398600}{10,000^3}\left(10,000\hat{\boldsymbol{x}}\right) = -0.004 \quad \frac{\text{km}}{s^2}$

   $$\mathbf{s} = \begin{bmatrix}10000\\0\\0\\0\\10\\0\end{bmatrix}\begin{matrix}\text{km}\\ \\ \\ \frac{\text{km}}{s}\end{matrix} \qquad \text{and} \qquad \dot{\mathbf{s}} = \begin{bmatrix}0\\10\\0\\-0.004\\0\\0\end{bmatrix}\begin{matrix}\frac{\text{km}}{s}\\ \\ \\ \frac{\text{km}}{s^2}\end{matrix}$$

4. Orbit 2 provides greater exposure to point A as a result of Kepler's second law.

5. $\varepsilon = -\frac{\mu}{2a}$ for elliptical orbits

   $\varepsilon_{ISS} = -\frac{\mu}{2(6378+422)} \qquad \varepsilon_{GPS} = -\frac{\mu}{2(6378+20,200)}$

   $\varepsilon_{ISS} < \varepsilon_{GPS}$

6. Orbital Period $T = 2\pi\sqrt{\frac{a^3}{\mu}}$

   $T_{ISS} = 2\pi\sqrt{\frac{a^3_{ISS}}{\mu}} = 2\pi\sqrt{\frac{(6378+422)^3}{\mu}}$

   $T_{GPS} = 2\pi\sqrt{\frac{a^3_{GPS}}{\mu}} = 2\pi\sqrt{\frac{(6378+20,200)^3}{\mu}}$

   $T_{ISS} < T_{GPS}$

7. False: The Earth and Mars are on two separate, distinct orbits of differing angular momentum, period, semi-major axis etc. Kepler's second law does not provide any insight regarding the relationship of the

Earth's orbital velocity to that of Mars.

8. True

9. False: $\frac{\mathrm{d}\boldsymbol{h}}{\mathrm{d}t} = 0$ does not imply $\|\boldsymbol{r}\| \|\boldsymbol{v}\|$ =constant

10. False: $\boldsymbol{r} \cdot \boldsymbol{v} = 0$ only at perigee, apogee, and for circular orbits