EML 5215

Exam 1

Fall 2015

16-17 October 2015

What Allowed During Examination

You may use any books, your personal notes, or electronic aid, provided that you find the material on your own without having it provided to you by anyone else (either implicitly or explicitly). You may not, under any circumstances, communicate with anyone about this exam.
Any violations of the exam rules will result in further action on my part in a manner consistent with the academic honesty policy of the University of Florida. The academic honesty policy can be found at the Student Conduct and Conflict Resolution website:
https://www.dso.ufl.edu/sccr/process/student-conduct-honor-code/

Guidelines for Solutions

Communication is an extremely important part of demonstrating that you understand the material. To this end, the following guidelines are in effect for all problems on the examination:

1. Your handwriting must be neat. I will not try to decipher sloppy handwriting and will assume that something is incorrect if I am unable to read your handwriting.

2. Your test must be HANDWRITTEN, no software, no scans, etc., your own handwriting ONLY. If anything else appears other than your own handwriting, the test will be evaluated at 0 (zero).

3. You must be crystal clear with every step of your solution. In other words, any step in a derivation or statement you write must be unambiguous (i.e., have one and only one meaning). If it is ambiguous as to what you mean in a step, then I will assume the step is incorrect.

4. Tests without name on each page, and/or without UFID and signature at the bottom of this page, will not be graded, i.e., they will count as a 0 (zero).

In short, please write your solutions in an orderly fashion so that somebody else can make sense of what you are doing and saying. Finally, credit will be given only if a relevant concept is applied properly, and no credit will be given for an incorrectly applied concept even if the final answer is correct.

University of Florida Honor Code

On your exam you must state and sign the University of Florida honor pledge as follows:

On my honor, I have neither given nor received unauthorized aid in doing this examination.

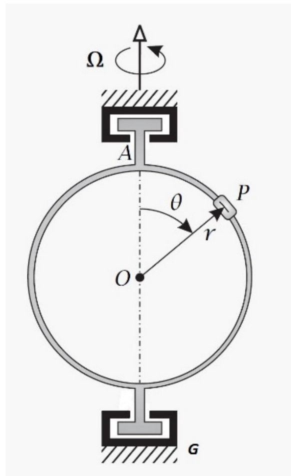
Signature: **Date:**

University of Florida ID:

Total points: 100

Question 1: 30 points

A collar of mass *m* slides on the rigid annulus (*A*) in the figure below, with viscous friction between *m* and the annulus (constant *C*). The annulus rotates with respect to the inertial ground (*G*) with constant angular velocity $\underline{\Omega}$, as depicted in the same figure. Find the equations of motion for the collar using the Angular Momentum approach.



Question 2: 20 points

Suppose that O, P, and Q are rigidly connected points in three-dimensional Euclidean space. Suppose further that $\underline{r}_{P/O}$, $\underline{r}_{Q/O}$, and $\underline{r}_{P/Q}$ are the vectors from O to P, O to Q, and Q to P, respectively.

(a) Show how the aforementioned information can be used to construct a right-handed orthonormal basis. **[15 points]**

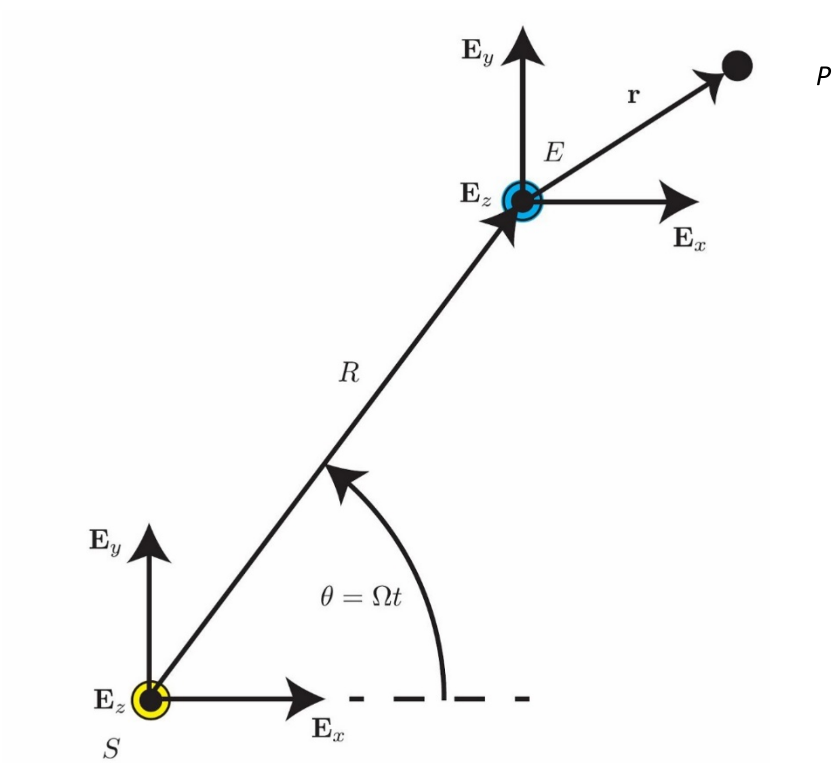
(b) Under what conditions does your solution to part (a) break down? **[5 points]**

Question 3: 25 points

Consider the following simplified two-dimensional model of a Sun-Earth-spacecraft system. The Sun is located at point *S*, the Earth is located at point *E*, and the spacecraft is located at point *P*. Suppose further that the coordinate system with origin *S* and basis {**Ex**; **Ey**; **Ez**} is inertially fixed and that the Earth does not rotate relative to the Sun. Furthermore, let the distance from the Sun to the Earth be the constant value *R*, while the angle between the direction **Ex** and the Sun-Earth line is $\theta = \Omega t$ (where Ω is constant). Finally, let the position vector of the spacecraft relative to the Earth be denoted **r**. Using this information, determine an expression for the acceleration of the spacecraft relative to the Sun in terms of the variables *x* and *y*, the constants *R* and Ω, and *t*, where *x* and *y* are the components of the position of the spacecraft relative to the Earth in the basis {**Ex**; **Ey**; **Ez**}.

Next, suppose that we use the following numerical values: $R = 1.49597891 \times 10^{11}\ m$ and $\Omega = 1.992 \times 10^{-7}\ rad/s$. In astronomical calculations, the Earth is often considered to be an inertial reference frame, and the coordinate system with the origin at the Earth and the basis {**Ex**; **Ey**; **Ez**} is called an Earth-centered inertial Cartesian coordinate system. Using the aforementioned numerical values, justify when it is reasonable to consider the Earth to be an inertial reference frame. Would the assumption that the Earth is an inertial reference frame be reasonable if the numerical values were $R = 100\ m$ and $\Omega = 1\ rad/s$? Why or why not?

<u>HINT</u> In your justifications use the following information/hint: 1) the acceleration of an orbiting object due to the Earth's attraction, as seen by an observer in the Earth-centered frame, is $\ddot{\mathbf{r}} = \left(-\mu/\|\mathbf{r}\|^3\right)\mathbf{r}$, $\mu = 3.9860 \cdot 10^{14}\ m^3/s^2$; 2) compute its magnitude at multiples of the radius of a geosynchronous orbit ($\|\mathbf{r}\| = 42{,}164\ km$).



Question 4: 25 points

The rigid wheel (W) of radius *r* in figure below rolls without slip on top of a flat rigid surface (G), and is also in contact with a top rigid surface, which is part of G (imagine the wheel moving in a pipe, for example). The two points of contact are denoted Q (bottom – where roll occurs) and D (top – where there is sliding). Find an expression for the viscous friction force acting on the wheel at point D. *C* is the coefficient of viscous friction.