

**EML 5215**

**Exam 2**

**Spring 2015**

**21-22 April 2015**

## What Allowed During Examination

You may use any books, your personal notes, or electronic aid, provided that you find the material on your own without having it provided to you by anyone else (either implicitly or explicitly). You may not, under any circumstances, communicate with anyone about this exam.

Any violations of the exam rules will result in further action on my part in a manner consistent with the academic honesty policy of the University of Florida. The academic honesty policy can be found at the Student Conduct and Conflict Resolution website:

<https://www.dso.ufl.edu/sccr/process/student-conduct-honor-code/>

## Guidelines for Solutions

Communication is an extremely important part of demonstrating that you understand the material. To this end, the following guidelines are in effect for all problems on the examination:

1. Your handwriting must be neat. I will not try to decipher sloppy handwriting and will assume that something is incorrect if I am unable to read your handwriting.
2. Your test should be HANDWRITTEN, no software or other tools, your own handwriting.
3. You must be crystal clear with every step of your solution. In other words, any step in a derivation or statement you write must be unambiguous (i.e., have one and only one meaning). If it is ambiguous as to what you mean in a step, then I will assume the step is incorrect.

In short, please write your solutions in an orderly fashion so that somebody else can make sense of what you are doing and saying. Finally, credit will be given only if a relevant concept is applied properly, and no credit will be given for an incorrectly applied concept even if the final answer is correct.

## University of Florida Honor Code

On your exam you must state and sign the University of Florida honor pledge as follows:

**On my honor, I have neither given nor received unauthorized aid in doing this examination.**

**Signature:**

**Date:**

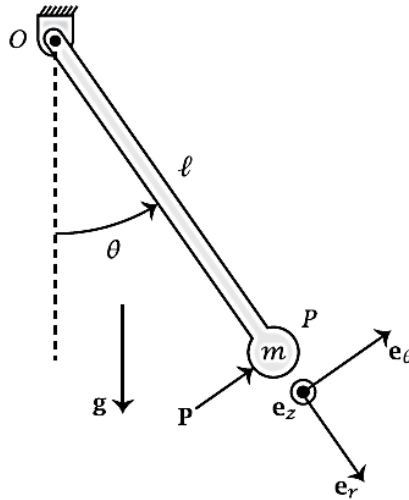
**University of Florida ID:**

**Total points: 100**

**Question 1: 20 points**

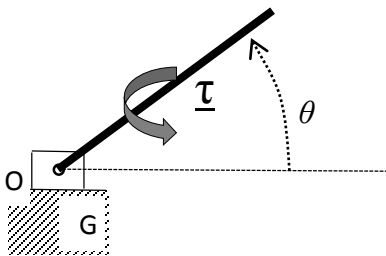
A particle of mass  $m$ , located at point  $P$ , is suspended from the end of a rigid massless arm of length  $\ell$ . The arm is hinged at the inertially fixed point  $O$ , and the radial, transverse, and normal directions to the arm are defined as  $\mathbf{e}_r$ ,  $\mathbf{e}_\theta$ , and  $\mathbf{e}_z$ , respectively. Knowing that gravity acts vertically downward and that the external force  $\mathbf{P} = P\mathbf{e}_\theta$  acts in the direction transverse to the rod, show the following:

- (i) The reaction force exerted by the arm on the particle has no component in the direction transverse to the arm.
- (ii) The generalized force  $Q_\theta$  corresponding to the generalized coordinate  $\theta$  is  $Q_\theta = -mg\ell \sin \theta + \ell P$ .



**Question 2: 20 points**

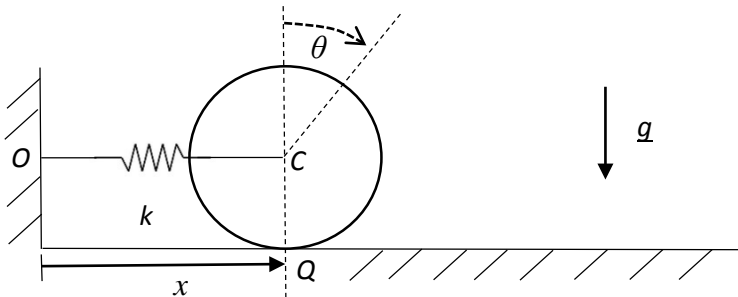
Using the definition of pure torque, find the generalized force for the homogeneous bar in the picture below. The bar's length is  $l$ . The vector  $\underline{\tau}$  is out of the page. The motion is planar in the page and  $O$  is an inertially fixed point. Use the expression  $Q_i = \sum_{j=1}^m \mathbf{F}_j \cdot \frac{\partial \mathbf{r}_j}{\partial q_i}$  where  $m$  is the number of forces acting on the rigid body.



**Question 3: 35 points**

A homogeneous disk  $D$  of radius  $r$  and mass  $m$  rolls without slip on a flat surface considered the ground  $G$  (inertial). A linear spring of constant  $k$  connects its center of mass with a wall fixed to the ground. The length of the spring at rest (uncompressed/un-stretched) is  $x_0$  and the angle  $\theta$  is 0 when  $x=x_0$ .  $O$  is an inertially fixed point,  $Q$  is the instantaneous point of contact,  $C$  is  $D$ 's center of mass. See the image below for more details.

Using Euler's Laws find the equation of motion for the disk in terms of the variable  $x$ .



**Question 4: 25 points**

Solve the problem in question 3 using Lagrange's Equations.

**OPTIONAL – COUNTED ONLY IF QUESTION 3 HAS BEEN COMPLETED (5 points)**

In implementing Euler's second Law, why do  ${}^G\mathbf{H}_Q$  and the second Law appear to be exactly the same if point  $Q$  is used as part of the ground or part of the disk?