Feedback Control of Spacecraft Rendezvous Maneuvers using Differential Drag

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This work presents a feedback control strategy to perform spacecraft rendezvous maneuvers exploiting differential drag forces. Differential drag is an alternative method for generating control forces at low Earth orbits, by varying the aerodynamic drag experienced by different spacecraft, thus generating differential accelerations between the vehicles, virtually without use of propellant. The variation in the drag can be induced by modifying the spacecraft cross-wind sectional area. The interest towards this methodology comes from the decisive role that efficient and autonomous spacecraft rendezvous maneuvering will have in future space missions, since virtually propellant-free maneuvers can be performed. The proposed approach controls the nonlinear dynamics of spacecraft relative motion using on-off control, by introducing a linear model to analytically generate a guidance trajectory, based on the linearized equations of spacecraft relative motion. A guidance control sequence trajectory is used for the control of the real dynamics until an error threshold is met. Once this occurs, a new guidance trajectory is generated, and its control sequence is used. The process is repeated until rendezvous conditions are satisfied. A numerical simulation is presented to validate the approach.

1. Introduction

This paper presents a feedback control strategy for the rendezvous maneuver of a chaser and a target spacecraft using aerodynamic differential drag. Control of space rendezvous maneuvers is an increasingly important topic, given the potential for its application in operations such as autonomous guidance of satellite swarms, on-orbit maintenance missions, refueling and autonomous assembly of structures in space. Several control strategies for spacecraft rendezvous maneuvers using thrusters have been developed in past few years (see references [1] and [2]). Nevertheless, given the high cost of refueling, an alternative for thrusters as the source of the control forces is desired with the intent to reduce costs for future space missions. A differential in the aerodynamic drag experienced by the target and chaser spacecraft produces a respective differential in acceleration between the spacecraft which can be used to control the motion of the chaser relative to the motion of the target spacecraft. The concept of spacecraft maneuvering using differential drag was first proposed by C.L.Leonard [3]. The main advantage of differential drag maneuvering is that it does not require the use of any type of propellant, which is clearly not the case for standard thruster-driven maneuvers. However, these maneuvers can only be performed at low Earth orbits, where there are enough atmospheric particles to generate sufficient drag forces. An application of these principles is the JC2Sat project being developed by the Canadian and Japanese Space Agencies (see references [4] and [5]).

The reference frame commonly employed for spacecraft relative motion representation is the Local Vertical Local Horizontal (LVLH) reference frame, where x points from Earth

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to the reference satellite (virtual or real), y points along the track (direction of motion), and z completes the right-handed frame (see Fig. 1).
To generate the drag differential, the chaser and target spacecraft must have different cross-wind sectional areas. A simple way of achieving this is to provide the spacecraft with a system of rotating flat plates which in practice can be solar panels. Three cases for the configurations (see Fig. 1) of the plates are considered, where a chaser spacecraft can increase, equal, or decrease its atmospheric drag with respect to a target. The plates re-orientation delay time is assumed negligible with respect to the maneuvers duration, thus, it is assumed that the plates rotate instantly, generating a bang-off-bang control profile, as suggested in [6], [7], and [8].

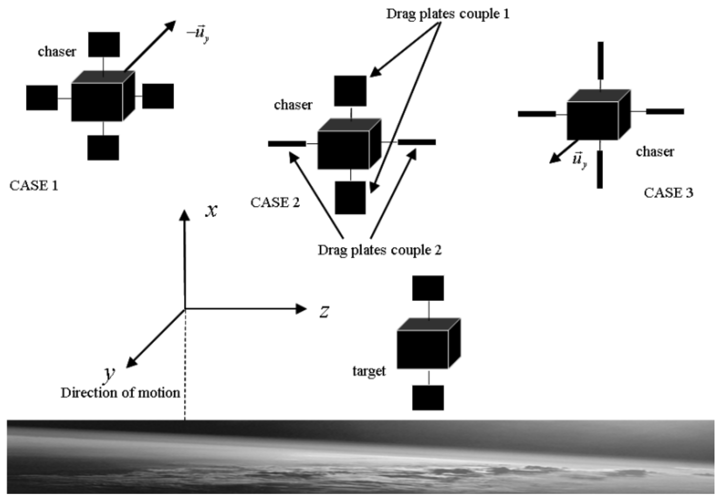

Fig. 1 Drag plates concept to generate differential drag (obtained from [8]).

In order to cope with the reality of nonlinearities and non modeled effects, a feedback approach for forcing nonlinear systems, controlled by on-off actuators only, to track linear models is here proposed.
The control strategy selects the sequence of positive, negative or zero differential accelerations of the chaser relative to the target. This selection is designed so that the dynamics of the spacecraft system (chaser and target spacecraft) tracks a linear reference model, whose utilization allows for analytical development of a guidance trajectory and its respective control sequence. This control sequence is applied to the real nonlinear dynamics of the system. As the maneuver progresses, the tracking error grows due to the difference in dynamical behavior between the linear model used to create the guidance and the nonlinear system. Once this error reaches a threshold, a new guidance trajectory is generated and its respective new control sequence is implemented. This procedure is repeated until rendezvous conditions are achieved. This significantly simplifies the control problem, since the desired trajectory (in this case the trajectory for the rendezvous maneuver) can be designed using linear control techniques on the linear reference model. Moreover, the use of this control method allows for the design of a smoother control action with less switching in the differential drag than previous work of one of the authors [8].
The foremost contributions in this work are:

1) A feedback control strategy for the rendezvous maneuver of a chaser and target spacecraft using differential drag.

2) Demonstration of feasibility of the approach via numerical simulations.
3) Assessment of the performances of the designed control strategy in terms of the accuracy in the tracking of the linear reference model and the number of switches in the differential drag caused by the control strategy.

This work is organized as follows: Section 2 presents the spacecraft system dynamics explaining the effect of aerodynamic drag, Section 3 illustrates the guidance used, Section 4 comments on the Nonlinear Model used, Section 5 explains the development of the Feedback Control Strategy, Section 6 contains the simulations performed, and finally, Section 7 presents the conclusions.

2. Relative Motion Spacecraft Dynamics

Hill's seminal paper on lunar theory [9] which described the motion of the moon relative to the Earth was the first study on the relative motion of bodies in space. Based on Hill's work, Clohessy and Wiltshire [10] developed the linear model that bears their name, which describes the motion of a chaser spacecraft relative to a target spacecraft. This model has been widely used in applications involving low thrust proximity maneuvers. However, this model does not account for the differential effects on the spacecraft motion due to nonlinearities such as the J_2 perturbation, caused by the Earth's flattening. The effect of the J_2 perturbation and other nonlinearities is more significant in maneuvers with longer times of execution such as those performed using differential drag. For this reason the use of a linear model that partially accounts for averaged effects of these nonlinearities is desired.

2.1. Schweighart and Sedwick Relative Motion Equations

A linearized model which represents the relative motion of spacecraft under the influence of the J_2 was developed by Schweighart and Sedwick [11]. By simply adding the control acceleration vector (**u**) to the Schweighart and Sedwick equations, the following system of linear differential equations in terms of relative positions, velocities and accelerations in the LVLH is obtained:

$$a_x - 2ncV_y - (5c^2 - 2)n^2 x = u_x \tag{1}$$

$$a_y + 2ncV_x = u_y \tag{2}$$

$$a_z + q^2 z = 2lqcos(qt + \varphi) + u_z \tag{3}$$

Where the c and n are defined as follows:

$$c = \sqrt[2]{1 + \frac{3J_2 R}{8{r_T}^2}[1 + 3cos(2i_t)]}, \quad n = \frac{\omega_R}{c} \tag{4}$$

This set of equations is used to generate the desired trajectory for the rendezvous maneuver. It is important to note that equations (1), (2), and (3) assume circular reference orbit, a small separation between target and chaser in comparison to the radii of their orbits, and that only J_2 effects, drag and 2-body forces are acting on the spacecraft system. Moreover, the Schweighart and Sedwick equations do not provide any information related to the attitude of the spacecraft, and in this paper it is assumed that attitude is stabilized by other means than the differential drag.

2.2. Differential Drag

At low Earth orbits (LEO) there is still a significant amount of atmospheric particles, which induces a pressure on the surface of any object moving at those orbits. In others words at LEO atmospheric density (ρ) is large enough to induce aerodynamic drag against the motion of a spacecraft. The relative acceleration caused by the differential aerodynamic drag for the spacecraft system is given in [7] as:

$$a_D = \frac{\rho \Delta BC}{2} V_r^2 \tag{5}$$

where the ballistic coefficient *ΔBC* is given by:

$$\Delta BC = \frac{m_T C_{DC} S_C - m_C C_{DT} S_T}{m_T m_C} \tag{6}$$

Since this acceleration is caused by drag, then it only acts in the direction opposite to motion (negative y direction in the LVLH frame). This means that the only nonzero component of the control vector is u_y. Hence the control vector in the LVLH frame is:

$$\boldsymbol{u} = [0 \ -a_D \ \ 0]^{\mathrm{T}} \tag{7}$$

It can be observed from equations (1), (2), and (3) that the dynamics in the x and y directions are independent of those on the z direction. Also from equation (7) it can be observed that control can be achieved only on the along-track direction of orbital motion (y direction). This indicates that by using differential drag, only motion in the x-y plane, with its velocities, can be controlled for the dynamics described in equations (1) and (2).

2.3. Transformation of the Schweighart and Sedwick equations

Taking into account only equations (1), and (2) the state space representation of the Schweighart and Sedwick equations is defined in equation (8). The motion of the spacecraft system outside of the x-y plane is not considered since the drag cannot influence it.

$$\dot{\boldsymbol{x}} = [\underline{\mathbf{A}}_1] + \begin{bmatrix} \underline{\mathbf{0}}_{2x2} \\ \underline{\mathbf{I}}_{2x2} \end{bmatrix} \boldsymbol{u} \tag{8}$$

with matrix $\underline{\mathbf{A}}_1$ and vector state **x** defined as follows:

$$\underline{\mathbf{A}}_1 = \begin{bmatrix} \multicolumn{2}{c}{\underline{\mathbf{0}}_{2x2}} & \multicolumn{2}{c}{\underline{\mathbf{I}}_{2x2}} \\ (5c^2-2)n^2 & 0 & 0 & 2nc \\ 0 & 0 & -2nc & 0 \end{bmatrix}, \ \boldsymbol{x} = \begin{bmatrix} x & y & V_x & V_y \end{bmatrix}^{\mathrm{T}} \tag{9}$$

The state space representation of the Schweighart and Sedwick equations can be decomposed into a double integrator and a harmonic oscillator as described in [12], resulting in a new state vector **z** using the following transformation:

$$\boldsymbol{z} = \begin{bmatrix} 0 & 1 & -a/d^2 & 0 \\ -ab/d^2 & 0 & 0 & -b/d^2 \\ 0 & 0 & a^2/2d^3 & 0 \\ -a^2b/2d^3 & 0 & 0 & a^3/2d^3 \end{bmatrix} \boldsymbol{x}, \qquad a = 2nc, b = (5c^2-2)n^2, d = \sqrt{a^2 - b} \tag{10}$$

The solution to the state space system (10) is analytical when the control is of bang-off-bang nature. It can be found in reference [7].

3. Guidance

Analitycal guidance for the rendezvous problem can be found by linearizing the dynamics of the spacecraft system resulting in the Schweighart and Sedwick equations, as described in the following.

3.1. Trajectory for the Schweighart and Sedwick model

The rendezvous maneuver can be preliminarily separated into two sequential phases, as previously suggested in the literature ([7] and [8]). In the first phase the spacecraft are driven towards a stable relative orbit; in the second phase the oscillation of the relative orbit is canceled out and the rendezvous conditions are achieved, namely zero relative position and velocity. In order to find this solution, the state vector transformation from the LVLH coordinate to the new set of coordinates $\mathbf{z} = [z_1\ z_2\ z_3\ z_4]^T$ appears to be convenient since it provides a representation of the system behavior in which the dynamics of the system are decoupled into a double integrator and a harmonic oscillator. The transformation is linear and it does not shift the origin, which means that to satisfy the rendezvous condition the transformed coordinates (**z**) must reach zero for rendezvous to occur.
In the transformed coordinate system the first phase of the maneuver drives the z_1 and z_2 components to zero. This process is demonstrated in Fig. 2 a) in which starting from state α, a differential acceleration between the vehicles is induced. Due to its bang-off-bang nature this differential acceleration can only be negative, positive or zero. The sign of this relative acceleration is changed at states β, γ, and δ thus driving z_1 and z_2 to the origin in the z_1-z_2 plane.
The second phase consists of driving z_3 and z_4 to zero. This results in z_1 and z_2 deviating from the origin in the z_1-z_2 plane; however, by applying the negative and positive relative accelerations, required to drive z_3 and z_4 to the origin in the z_3-z_4 plane, for exactly the same time intervals, the z_1 and z_2 coordinates will reach the origin in the z_1-z_2 plane at the end of the second phase. The second phase, only for the z_3-z_4 plane, is illustrated in Fig. 2 b) in which, starting from state ε no relative acceleration is applied until state ς reached. This inactive period of Δt_w is desired in order to assure that the system follows a trajectory in which the time interval for positive and negative relative accelerations is the same. As shown in [8] the value of Δt_w can be found as follows:

$$\Delta t_w = \frac{1}{d}\left|\tan^{-1}\left(\left.\frac{z_3}{z_4}\right|_{\varsigma}\right) - \tan^{-1}\left(\left.\frac{z_3}{z_4}\right|_{\varepsilon}\right)\right| \tag{11}$$

Afterwards, starting from state ς a relative acceleration is again induced. Then at states η and θ the sign of the relative accelerations is changed after time intervals of Δt and 2Δt respectively. As shown by [7] this time interval Δt can be calculated using:

$$\Delta t = \frac{1}{d}\cos^{-1}\left[\frac{1}{2}\left(1+\sqrt{h}-\sqrt{3-h-\frac{2}{\sqrt{h}}}\right)\right] \tag{12}$$

where coefficients h, f, and g are given as:

$$h = 1 + \frac{\sqrt[3]{f}}{6g} - \frac{e_0}{g\sqrt[3]{f}},\ f = -54ge_0^2 + 6\sqrt{3}e_0^2\sqrt{2e_0^2 + 27g^2},\ g = -\frac{\sqrt{2}a^3\left|u_y\right|}{2d^5}i \tag{13}$$

Subsequently, the sign of the relative acceleration is maintained for another Δt time interval after which the origin in the z_3-z_4 plane is reached. Since positive and negative relative accelerations are maintained for net time intervals of 2Δt, the origin is reached in both z_1-z_2 and z_3-z_4 planes. Once the trajectory for the two phases in the transformed coordinates is found, the analytical control sequence is translated back into the original x, y reference, and the guidance trajectory is generated.

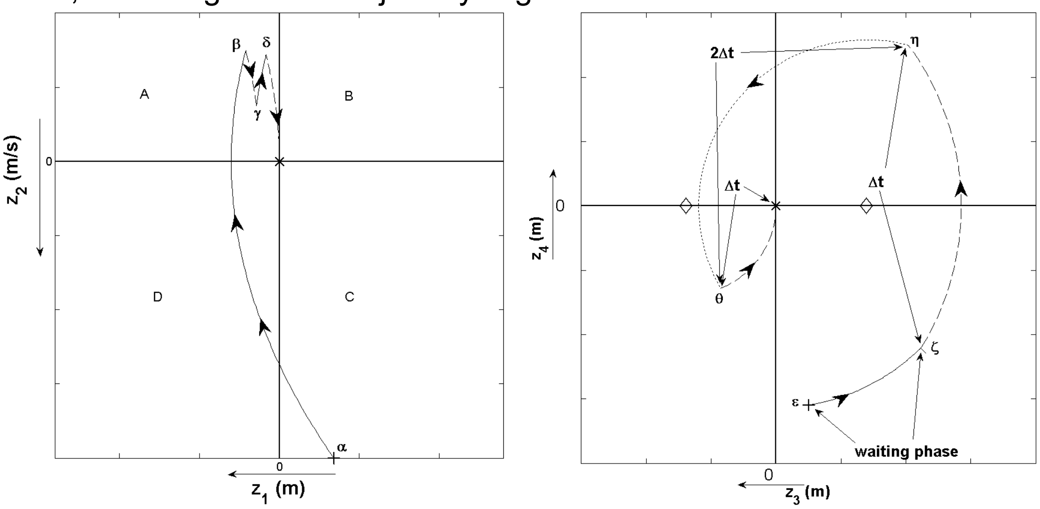

Fig. 2 Two-phase differential drag guidance ([7]): a) first phase: stabilization of the relative orbit, b) second phase: final rendezvous, canceling out oscillations.

Furthermore the time-varying eccentricity of the harmonic oscillator motion before rendezvous is given as:

$$e_0 = \sqrt{z_3^2 + \left(\frac{z_4}{d}\right)^2} \tag{14}$$

As indicated in [7] at the end of the first phase e_0 must be smaller than e_c otherwise the inverse cosine in equation (12) cannot be evaluated. This critical value was determined in [7] to be:

$$e_c = \frac{13a^3\left|u_y\right|}{5d^5} \tag{15}$$

As recommended in [7] if at the end of the first phase e_0 is larger than e_c then e_0 is corrected using the following equation:

$$e_0 = e_0 - 0.99e_c \tag{16}$$

And then equation (12) is used and phase two is performed as illustrated before. The second phase is repeated until e_0 is larger than e_c and then the final second phase is executed driving all the **z** states to zero. It is important to note that in a real-world application of this guidance the tracking error is not expected to reach zero, but to reach a residual value near the origin. The reason for this behavior is the on-off nature of differential drag which does not allow for a smooth control action.

The real world problem of designing a control system for the rendezvous maneuver using differential drag becomes the problem of designing a feedback control law for the flat panels, forcing the satellites to follow the two-phase guidance (see Fig. 2), coping with nonlinearities, uncertainties, and navigation errors. Previous results [7] suggest control implementations to track the guidance shown in Fig. 2, that results in satisfactory differential drag based rendezvous maneuvers. However, some thrusting capability was required to complete the mission in previous work [7], due to small inaccuracies in guidance tracking, and a high number of switching commands to the drag plates. These limitations translate into a non propellant-free maneuver, and demanding energy requirements for the plates' actuators.

4. Nonlinear Model

The dynamics of spacecraft relative motion are nonlinear due to effects such as the J_2 perturbation. The Feedback approach here suggested intends to cope with these limitations. In this section the model used for the nonlinear dynamics and for testing the control strategy is presented. The general expression for the real world nonlinear dynamics of the acceleration of a spacecraft is defined as:

$$\boldsymbol{a} = \frac{-\mu \boldsymbol{r}}{r^3} + \boldsymbol{u} + \boldsymbol{a}_{J2} \tag{17}$$

where **u** is the control acceleration caused by aerodynamic drag in the inertial frame, and $\mathbf{a}_{J2}$ is the acceleration caused by the J_2 perturbation defined in the inertial frame as:

$$\boldsymbol{a}_{J2} = -\frac{3}{2}\frac{J_2 R_e^{\,2}}{r^4}\begin{bmatrix} (1-3\frac{z^2}{r^2})\frac{x}{r} & (1-3\frac{z^2}{r^2})\frac{y}{r} & (3-3\frac{z^2}{r^2})\frac{z}{r} \end{bmatrix}^T \tag{18}$$

5. Feedback Control Strategy

In this section the feedback control law is designed, with the aim of mitigating the current limitations of differential drag control. A guidance trajectory following the logic described in section 3 is generated such that it drives the linear system from its initial state (initial positions and velocities of the target and chaser spacecraft) to the rendezvous state. Along with this guidance trajectory, the control sequence for the linear model to follow this trajectory is also generated. This control sequence is applied to the nonlinear model. Since the dynamical behavior of the two models diverges over time, the tracking error grows as the maneuver is propagated with the nonlinear dynamics. For the suggested approach the tracking error in velocity only was used. Once the tracking error reaches a critical value, a new guidance trajectory that drives the linear system from the current state to the rendezvous state is recomputed, along with its control sequence. This new control sequence is implemented on the nonlinear model, and the process is repeated until the state of the nonlinear model reaches rendezvous conditions. The critical value for the tracking error is reduced as the state of the system approaches rendezvous conditions. It is worth emphasizing that all the components in the control strategy described above would be available in real time, and therefore on board the spacecraft.

6. Numerical Simulations

Numerical simulations, using the parameters shown in Table 1, and the initial conditions shown in Table 2 (previously used in [7]), were used to validate the approach.

Table 1 Simulation Parameters

Parameter	Value
J_2	1.08E-03
R (km)	6378.1363
i_T (deg)	51.595
r_T (km)	6728.1363
μ(km^3/sec^2)	398600.4418
V_r (km/sec)	7.68
m_T (kg)	10
m_C (kg)	11
S_T (km^2)	5.0E-07
S_C (km^2)	3.0E-06
C_{DT}	2
C_{DC}	5

Table 2 Chaser initial position, and velocity in the LVLH

Parameter	Value
x (km)	-1
y (km)	2
V_x (km/sec)	-8.40E-06
V_y (km/sec)	-1.70E-04

For the simulations, a MATLAB program that generates the guidance trajectories and their respective control sequences was created. The nonlinear dynamics of the system were propagated using Simulink and, as explained before, new guidance trajectories were generated each time the tracking error reached the critical value. For the Initial conditions and parameters chosen, it was necessary to generate six linear guidance trajectories in order for the system to approximate rendezvous conditions. The resulting simulated trajectories in the x-y and Vx-Vy planes of the nonlinear system along with the initial guidance trajectory can be seen in Fig. 4.

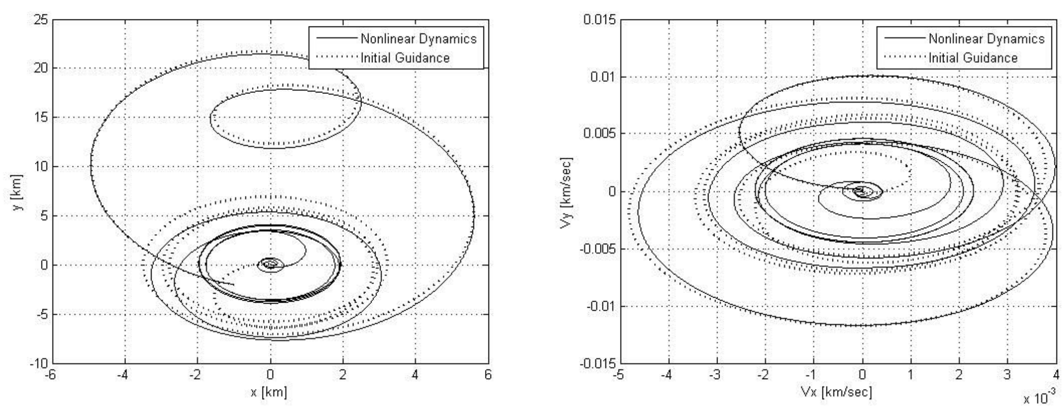

Fig. 3 Simulated Trajectory in the x coordinates: a) x-y plane, b) V_x- V_y plane

Furthermore the final stages of the rendezvous maneuver simulation can be seen in Fig. 4. The final relative distance and velocity between the Chaser and the Target were of 28 m and 1.647 cm/s respectively.

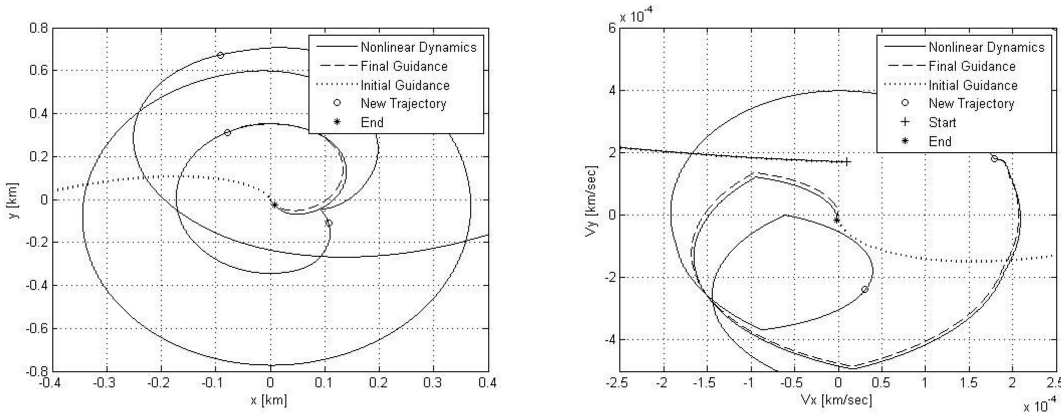

Fig. 4 Detail of the Simulated Trajectory: a) x-y plane, b) V_x- V_y plane

The control profile over the entire rendezvous maneuver (see Fig. 5) requires a low frequency of actuation in comparison with previous results [7], which allows for realistic considerations for the application of the feedback control presented here.

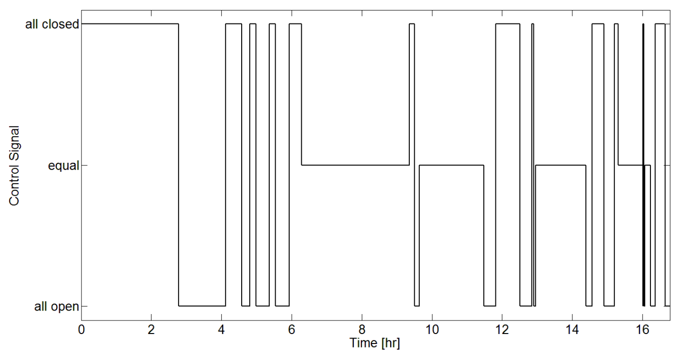

Fig. 5 Control signal profile over the entire maneuver (see Fig. 1).

7. Conclusion

In this work a new application of the of feedback principles for the autonomous control of a spacecraft rendezvous maneuver by making use of differential drag is presented. By varying the differential drag between the chaser, and target satellites their motion in the x-y and V_x-V_y planes in the LVLH frame is controlled. These variations are induced by the action of sets of plates fixed to the spacecraft. The nature of this variation is assumed to be of bang-off-bang nature with only three possible values: maximum differential acceleration, minimum differential acceleration, and zero differential acceleration. Trajectories of the rendezvous maneuver are generated using the Schweighart and Sedwick linear model and implemented on a nonlinear model of the system. Once the tracking error reaches a critical value, a new guidance trajectory is generated and implemented. The process is iterated until the system reaches the rendezvous state. The resulting controlled actuation is an improvement over previous results since it presents a significantly lower frequency of actuation and a reasonably small residual error.

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