# Spacecraft De-Orbit Point Targeting using Aerodynamic Drag 

Sanny R. Omar ${ }^{1}$ and Riccardo Bevilacqua ${ }^{2}$<br>University of Florida, ADAMUS Laboratory, 939 Sweetwater Dr., Gainesville, FL 32611


#### Abstract

The ability to re-enter the atmosphere at a desired location is important for spacecraft containing components that may survive re-entry. This paper discusses the use of solely aerodynamic drag force to perform this targeting, which is especially useful for smaller spacecraft that do not contain thrusters. It is shown that by varying the ballistic coefficient of a spacecraft over time, any desired longitude and latitude can be targeted provided that the maneuvering begins early enough and the latitude is less than the inclination of the orbit. An analytical solution based on perturbations from a numerically propagated trajectory is developed to estimate the ballistic coefficient profile necessary to reach a given target point assuming small deviations from the numerical trajectory. An iterative process whereby analytical solutions are tested and refined can be utilized to determine the ballistic coefficient necessary for re-entry point targeting. Monte Carlo simulations were conducted to validate the algorithm and the desired de-orbit points were reached within a tolerable error in all tested scenarios. The High Precision Orbit Propagator in AGI's Systems Tool Kit software was also utilized to validate the targeting algorithm.


## Nomenclature

| A | $=$ satellite area $\left[m^{2}\right]$ |
| :---: | :---: |
| $a$ | $=$ semi major axis [km] |
| $a_{d}$ | $=$ acceleration due to aerodynamic drag $\left[\mathrm{m} / \mathrm{s}^{2}\right]$ |
| $a_{0}$ | $=$ reference semi major axis for exponential atmospheric model [km] |
| $a_{i}$ | $=$ initial semi major axis in maneuver [ km ] |
| $a_{f}$ | $=$ final semi major axis in maneuver [ km$]$ |
| $a_{\text {swap }}$ | $=$ semi major axis at which ballistic coefficient is changed [km] |
| $C_{b}$ | $=$ ballistic coefficient [ $\mathrm{m}^{2} / \mathrm{kg}$ ] |
| $C_{b 1}$ | $=$ ballistic coefficient from $t_{0}$ to $t_{\text {swap }}$ in current maneuver $\left[\mathrm{m}^{2} / \mathrm{kg}\right]$ |
| $C_{b 10}$ | $=$ ballistic coefficient from $t_{0}$ to $t_{\text {swap }}$ in the initial trajectory $\left[\mathrm{m}^{2} / \mathrm{kg}\right]$ |
| $C_{b 20}$ | $=$ ballistic coefficient from $t_{\text {swap }}$ until terminal point in the initial trajectory $\left[\mathrm{m}^{2} / \mathrm{kg}\right]$ |
| $C_{b 2}$ | $=$ ballistic coefficient from $t_{\text {swap }}$ until terminal point in current maneuver [ $\left.\mathrm{m}^{2} / \mathrm{kg}\right]$ |
| $C_{b_{\text {avg }}}$ | $=$ average ballistic coefficient [ $\left.\mathrm{m}^{2} / \mathrm{kg}\right]$ |
| $C_{b_{\text {term }}}$ | $=$ ballistic coefficient from $t_{\text {term }}$ to $t_{\text {deorbit }}\left[m^{2} / \mathrm{kg}\right]$ |
| $C_{d}$ | $=$ drag coefficient [no units] |
| $e$ | $=$ eccentricity [no units] |
| $F_{r}$ | $=$ non-Keplerian acceleration in the radial direction $\left[\mathrm{km} / \mathrm{s}^{2}\right]$ |
| $F_{s}$ | $=$ non-Keplerian acceleration in the along-track direction $\left[\mathrm{km} / \mathrm{s}^{2}\right]$ |
| $g$ | $=$ acceleration due to gravity $\left[\mathrm{km} / \mathrm{s}^{2}\right]$ |
| $h$ | $=$ angular momentum [ $\left.\mathrm{km}^{2} / \mathrm{s}\right]$ |
| H | $=$ atmosphere scale height [km] |
| $i$ | $=$ orbit inclination [ rad ] |
| $k$ | $=$ Boltzmann's Constant [J/K] |
| $m$ | $=$ satellite mass [kg] |

[^0]American Institute of Aeronautics and Astronautics

| $m_{w}$ | $=$ average molecular weight of an atmospheric particle $[\mathrm{kg} / \mathrm{molecule}]$ |
| :--- | :--- |
| $n$ | $=$ mean motion $[\mathrm{rad} / \mathrm{s}]$ |
| $p$ | $=$ semi latus rectum $[\mathrm{km}]$ |
| $r$ | $=$ orbital radius $[\mathrm{km}]$ |
| $R_{e}$ | $=$ radius of the Earth $[\mathrm{km}]$ |
| $T$ | $=$ Temperature $[\mathrm{K}]$ |
| $t_{\text {deorbit }}$ | $=$ time at which the satellite deorbits $[\mathrm{s}]$ |
| $t_{s w a p}$ | $=$ time at which the ballistic coefficient is changed from $C_{b 1}$ to $C_{b 2}[\mathrm{~s}]$ |
| $t_{s_{\text {old }}}$ | $=$ swap time in the previous trajectory $[\mathrm{s}]$ |
| $t_{s_{n e w}}$ | $=$ swap time in the new trajectory $[\mathrm{s}]$ |
| $t_{\text {term }}$ | $=$ time at which the satellite enters the terminal phase of its trajectory $[\mathrm{s}]$ |
| $v$ | $=$ orbital velocity $[\mathrm{km} / \mathrm{s}]$ |
| $v_{r e l}$ | $=$ satellite velocity relative to free stream $[\mathrm{km} / \mathrm{s}]$ |
| $\Delta \theta_{10}$ | $=$ change in true anomaly from $t_{0}$ until $t_{s w a p}$ in the initial trajectory $[\mathrm{rad}]$ |
| $\Delta \theta_{20}$ | $=$ change in true anomaly from $t_{s w a p}$ until $t_{\text {term }}$ in the initial trajectory $[\mathrm{rad}]$ |
| $\Delta \theta_{1}$ | $=$ change in true anomaly from $t_{0}$ until $t_{s w a p}$ in the new trajectory $[\mathrm{rad}]$ |
| $\Delta \theta_{2}$ | $=$ change in true anomaly from $t_{s w a p}$ until $t_{\text {term }}$ in the new trajectory $[\mathrm{rad}]$ |
| $\Delta \theta_{t}$ | $=$ desired total change in true anomaly of new trajectory until the terminal point $[\mathrm{rad}]$ |
| $\Delta \theta_{d}$ | $=$ difference in total change in true anomaly between the new trajectory and the initial trajectory $[\mathrm{rad}]$ |
| $\Delta t_{10}$ | $=$ time until swap point (equivalent to $\left.t_{s w a p}\right)$ in initial trajectory $[\mathrm{s}]$ |
| $\Delta t_{20}$ | $=$ time from the swap point until terminal point in the initial trajectory $[\mathrm{s}]$ |
| $\Delta t_{1}$ | $=$ time until swap point (measured from simulation epoch) in the new trajectory $[\mathrm{s}]$ |
| $\Delta t_{2}$ | $=$ time from swap point until the terminal point in the new trajectory $[\mathrm{s}]$ |
| $\Delta t_{t}$ | $=$ total time the new trajectory takes to reach the terminal point $[\mathrm{s}]$ |
| $\Delta t_{d}$ | $=$ difference in total orbit lifetime between the new trajectory and the initial trajectory $[\mathrm{s}]$ |
| $\Delta C_{b}$ | $=$ term used to correlate $C_{b 1}$ and $C_{b 2}[$ no units] |
| $\theta$ | $=$ true anomaly $[\mathrm{rad}]$ |
| $\phi$ | $=$ angle between ascending node and spacecraft position measured along orbital track $[\mathrm{rad}]$ |
| $\mu$ | $=$ Earth's gravitational parameter $\left[\mathrm{km}{ }^{3} / \mathrm{s}^{2}\right]$ |
| $\rho$ | $=$ ambient density $\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ |
| $\rho_{0}$ | $=$ density at reference altitude $\left[\mathrm{kg} / \mathrm{m}^{3}\right]$ |
| $\Omega$ | $=$ right ascension of the ascending node $[\mathrm{rad}]$ |
| $\omega$ | $=$ argument of the periapsis $[\mathrm{rad}]$ |
| $\omega_{a v g}$ | $=$ average orbit angular velocity $[\mathrm{rad} / \mathrm{s}]$ |
| $\omega_{e}$ | $=$ rotation rate of Earth $[\mathrm{rad} / \mathrm{s}]$ |

## I. Introduction

ENSURING that space vehicles cause no damage to persons or property after de-orbit has been an important consideration since the beginning of the space program ${ }^{1,2}$. Most large space vehicles containing thrusters execute a precise de-orbit burn to initiate the re-entry trajectory ${ }^{3}$. Because the time required for this burn is generally short compared to the duration of the decay trajectory, it can be treated as a nearly instantaneous impulse (impulsive burn), making it relatively simple to calculate the burn's effects on the spacecraft's decay trajectory. Additionally, because de-orbit generally occurs within one or two orbits after the de-orbit burn, the re-entry guidance can be computed on the ground and uplinked to the spacecraft, leaving the spacecraft navigation and control system responsible only for tracking the precomputed guidance (usually through variation of the spacecraft's lift to drag ratio) ${ }^{4,5}$. In recent years, the miniaturization of technology has brought about small spacecraft such as CubeSats ${ }^{6}$ that may not contain thrusters or attitude control systems, and generally do not perform active re-entry control. These satellites have generally been built by universities ${ }^{7}$ and small organizations as teaching tools or testbeds for low-cost scientific experiments or technology demonstrations. As such, benign materials are generally used, and most components of the spacecraft are destroyed during re-entry and pose no threat to ground assets. However, there currently is an increasing demand for small satellites capable of performing advanced missions including Earth imaging, commercial communications, and astronomical observations. Performing these missions sometimes requires heavy metals or other materials that do not vaporize on re-entry ${ }^{1}$ which may cause a hazard to people on the ground. Satellites containing such materials may not be allowed to launch unless they have a means of guaranteeing safe and controlled re-entry.

American Institute of Aeronautics and Astronautics

If a propulsion system is not an option (due to cost or volume constraints), changing the aerodynamic drag the spacecraft experiences through modulation of the ballistic coefficient presents itself as the most feasible way to perform re-entry targeting. While extensive work has been done on density modeling and spacecraft drag estimation ${ }^{8,9}$, and there is a body of research focused on relative spacecraft maneuvering using differential drag ${ }^{10-12}$, very little research has been conducted on a de-orbit algorithm that utilizes solely aerodynamic drag to control the decay trajectory. Though the modulation of vehicle aerodynamics has been utilized since the Apollo missions ${ }^{4}$ to help vehicles maintain a precomputed guidance during the re-entry trajectory, the use of solely aerodynamic drag for reentry targeting presents a much greater challenge and has never been done in practice before to the best of the authors' knowledge. This is likely because the procedure is difficult, requires computing power beyond what was available on legacy space vehicles, and because until recently, there has not been a significant class of thruster-less spacecraft demanding such an algorithm. Because drag force is small and only acts in the retrograde direction, maneuvering must begin several days before the expected re-entry. If maneuvering is not initiated early enough, it may not be possible to target the desired longitude and latitude as will be demonstrated in the section IV (Controllability Analysis). Additionally, uncertainties in atmospheric density and spacecraft ballistic coefficients will significantly impact the decay trajectory because the errors propagate for multiple days. This and the inability to control out of plane motion using aerodynamic drag mean that a periodic re-computation of the guidance trajectory onboard the spacecraft will likely be required.

The most recent targeted de-orbit algorithm is published in Ref. 13, and is based on a two-phase decay trajectory. During the first phase, the satellite maintains a ballistic coefficient of $C_{b 1}$ until some semi major axis $a_{s w a p}$ is reached. After this point (phase 2), the satellite maintains a ballistic coefficient of $C_{b 2}$. This algorithm utilizes an analytical solution to create a mapping from the initial conditions and control parameters ( $C_{b 1}, C_{b 2}$, and $a_{s w a p}$ ) to the re-entry point. The mapping is then utilized to analytically calculate the control parameters needed to target a desired location. The analytical solution, however, presents some limitations. For one, the analytical solution requires an exponential atmospheric model with density given as

$$
\begin{equation*}
\rho=\rho_{0} \exp \left[-\frac{a-a_{0}}{H}\right] \tag{1}
\end{equation*}
$$

If the orbit is assumed circular, the total time and change in true anomaly that occur as a satellite under the influence of aerodynamic drag decays from one semi major axis to another is given by the equations

$$
\begin{gather*}
\Delta t=\int_{a_{0}}^{a_{f}}-\frac{d a}{2 \sqrt{\mu a} C_{b} \rho}  \tag{2}\\
\Delta \theta=\int_{a_{0}}^{a_{f}} \frac{d a}{2 C_{b} a^{2} \rho} \tag{3}
\end{gather*}
$$

Equations (2) and (3) are then integrated analytically after substituting Eq. (1) for $\rho$. This yields

$$
\begin{gather*}
\Delta t=\frac{\sqrt{\pi H}}{-2 C_{b} \sqrt{\mu} \rho_{0} e^{\frac{a_{0}}{H}}}\left[\operatorname{erfi}\left(\sqrt{\frac{a_{f}}{H}}\right)-\operatorname{erfi}\left(\sqrt{\frac{a_{i}}{H}}\right)\right]  \tag{4}\\
\Delta \theta=\frac{-1}{-2 C_{b} \rho_{0} H e^{\frac{a_{0}}{H}}}\left[\frac{a_{f} E i\left(a_{f} / H\right)-H e^{a_{f} / H}}{a_{f}}-\frac{a_{i} E i\left(a_{i} / H\right)-H e^{a_{i} / H}}{a_{i}}\right] \tag{5}
\end{gather*}
$$

where erfi and Ei are the imaginary error function and exponential integral respectively. The changes in time and true anomaly during phases one and two can be combined to calculate the total orbit lifetime and total change in true anomaly which can be used to calculate the impact location. This approach presents several issues. First of all, the exponential integral and imaginary error function become large within the range of possible input values, leading to significant truncation errors during practical computations. The algorithm also cannot be employed if the atmospheric model is not exponential, because there would be no closed form solution to the integrals in Eq. (1) and Eq. (2). A non-exponential density profile could be broken into bands where the density in each band increases roughly exponentially, but this still introduces some error. In addition, because the Earth is not a perfect sphere, $J_{2}$ perturbations will cause significant oscillations in altitude and semi major axis over the course of an orbit and the assumptions that density is a function of semi major axis and that semi major axis is monotonically decreasing will not hold, leading to a breakdown of the assumptions used to derive Eq. (1) and Eq. (2).

To simplify the process of calculating the control parameters needed for re-entry targeting, the authors of Ref. ${ }^{13}$ define $\Delta C_{b}$ such that

$$
\begin{align*}
& C_{b 1}=C_{b_{a v g}}\left(1+\Delta C_{b}\right)  \tag{6}\\
& C_{b 2}=C_{b_{a v g}}\left(1-\Delta C_{b}\right) \tag{7}
\end{align*}
$$

American Institute of Aeronautics and Astronautics


Figure 1. Targeting Error for Various Combinations of Swap Time and Delta $\mathrm{C}_{\mathrm{b}}$
$\Delta C_{b}$ and $a_{\text {swap }}$ now become the control parameters utilized for targeting. Once these parameters are estimated using the analytical solution, they are sent to a numerical optimizer. Unfortunately, this problem has numerous combinations of control parameters that act as local minimizers of the error between the desired and actual re-entry locations. This is illustrated in Figure 1 which displays the targeting errors that result from various control parameter values for an initial 300 km circular orbit with $C_{b_{\text {avg }}}=.0343 \frac{\mathrm{~m}^{2}}{\mathrm{~kg}}$.
Note that $t_{\text {swap }}$ is utilized instead of $a_{\text {swap }}$ to mark the point where the ballistic coefficient transitions from $C_{b 1}$ to $C_{b 2}$. Thus, especially since the error in the analytical solution may be large, there is no guarantee that the numerical optimization scheme will converge to the control parameters that globally minimize the targeting error. Additionally, while using the semi major axis as the condition for switching the ballistic coefficient may work in the simulator, the semi major axis will oscillate in reality due to the non-uniform gravitational field of the Earth and other environmental perturbations such as solar gravity, lunar gravity, and solar radiation pressure. This is the case even if mean orbital elements are used as most mean orbital element sets only average out the short term periodic effects of $J_{2}$ perturbations.

Despite these challenges, the increasing power of modern computers and the development of a drag device by the University of Florida Advanced Autonomous Multiple Spacecraft (ADAMUS) laboratory make targeted de-orbit using solely aerodynamic drag feasible ${ }^{14,15}$. This paper discusses an algorithm by which the ballistic coefficient of a spacecraft can be varied to achieve a desired spacecraft re-entry location. With this re-entry control scheme, a spacecraft capable of modulating its ballistic coefficient, whether through changes in attitude or deployment of a drag device, can safely re-enter away from populated areas. This control scheme could also enable spacecraft to fly through particular regions of the atmosphere during the decay trajectory if the satellite operators wished to conduct scientific observations in those regions. Satellites or re-entry vehicles containing thrusters could also utilize this control scheme (provided that they have a means of varying their ballistic coefficient) in order to conserve fuel during re-entry.

The algorithm proposed in this paper offers improvements over the algorithm in Ref. 13. The new algorithm likewise divides the decay trajectory into two phases and analytically maps initial conditions and control parameters to de-orbit location. In this case, the control parameters will be $C_{b 1}, C_{b 2}$, and $t_{s w a p}$. However, instead of relying on the assumption of a perfectly circular orbit and an exponential atmosphere, this analytical solution is based on perturbations from a numerically propagated trajectory. To generate an initial numerical trajectory, the spacecraft orbit is propagated from its initial conditions with some $C_{b 1}, C_{b 2}$, and $t_{s w a p}$. For small changes in the control parameters, the density vs. attitude profile and the velocity profile of the spacecraft remain similar to those of the initial numerical trajectory, and an analytical solution can be utilized to predict where the spacecraft will re-enter based on the initial trajectory without having to propagate another entire trajectory. This makes characterization of the perturbed trajectory almost instantaneous, simplifying the calculation of the control parameters needed to target a re-entry point. Additionally, because the initial trajectory is numerically propagated, any atmospheric model can be used (including advanced models such as NRLMSISE and JB2008) and effects of Earth's oblateness and the rotation of the atmosphere can also be added. Optionally, to increase convergence rate and minimize re-entry location error, an exponential altitude model can be created based on the numerically propagated trajectory and utilized for future iterations. The error in this exponential density model would be low since it would be based on the density encountered at each point in the numerical trajectory. Furthermore, the effects of noncircular orbits, variations in density, and environmental perturbations such as solar and lunar gravity are captured in the propagation of the numerical trajectory, so the errors in the analytical solutions shrink as the perturbations in the control parameters become less.

Section II discusses the analytical mapping from the initial state and control parameters to de-orbit location. Section III discusses a means of using this mapping to analytically determine a set of control parameters that reduce latitude and longitude targeting errors to the greatest extent possible. The procedure in this section can be applied iteratively as shown in Figure 2 until the control parameters needed for re-entry point targeting are calculated.


Figure 2. Simplified Targeting Algorithm Schematic
The analytical mapping from Section II facilitates a controllability analysis which is discussed in Section IV. Section V discusses the MATLAB high fidelity simulation environment utilized to create the numerically propagated trajectories. Section VI then presents the results of a Monte Carlo simulation campaign testing the targeting algorithm with a set of randomized initial conditions and desired re-entry locations. The validation of the targeting algorithm and the MATLAB propagator using AGI's Systems Tool Kit ${ }^{16}$ software is also discussed.

The use of analytical solutions makes the targeting algorithm feasible to run onboard a spacecraft with limited computational power, especially if additional code optimization is performed and the code is re-written in a faster language such as c++. Additionally, the analytical solutions allow for a detailed view of the effects of changes in certain control parameters on the re-entry location.

## II. Analytical Mapping from Initial State to Impact Location

The first step in this algorithm is to propagate an initial trajectory. Based on this initial trajectory, the impact location of a satellite with the same initial conditions but a different ballistic coefficient profile can be calculated analytically.

## A. Analyzing Effects of Orbital Perturbations

In order to use analytical techniques to calculate the de-orbit location, some relations between the changes in the orbital elements and the aerodynamic drag force must be developed. Assuming that the orbit is roughly circular facilitates the development of these relations. The Gaussian Variation of Parameters equations from Eq. 9-24 in Vallado's book ${ }^{17}$ give the change in semi major axis over time as

$$
\begin{equation*}
\frac{d a}{d t}=\frac{2}{n \sqrt{1-e^{2}}}\left[e \sin \theta F_{R}+\frac{p}{r} F_{s}\right] \tag{8}
\end{equation*}
$$

where $p$ is the orbit semi latus rectum, $F_{R}$ is the force in the radial direction, and $F_{s}$ is the force in the direction of the cross product between the angular momentum and orbit radius vectors. For a circular orbit around a spherical Earth, $e=0, p=r$, and $F_{s}=a_{d}$ and so Eq. (8) simplifies to

$$
\begin{equation*}
\frac{d a}{d t}=\frac{2 a_{d}}{n} \tag{9}
\end{equation*}
$$

For a circular orbit, mean motion is equal to instantaneous angular velocity and is given by the equation

$$
\begin{equation*}
n=\sqrt{\frac{\mu}{a^{3}}} \tag{10}
\end{equation*}
$$

Acceleration due to drag in a non-rotating atmosphere is given by the equation

$$
\begin{equation*}
a_{d}=-C_{b} \rho v^{2} \tag{11}
\end{equation*}
$$

where

$$
\begin{equation*}
C_{b}=\frac{C_{d} A}{2 m} \tag{12}
\end{equation*}
$$

Substituting Eqs. (10)-(12) into Eq. (9) and rearranging yields

$$
\begin{equation*}
-\sqrt{\frac{\mu}{a^{3}}}\left(\frac{1}{2 C_{b} \rho v^{2}}\right) d a=d t \tag{13}
\end{equation*}
$$

The velocity in a circular orbit is given by

$$
\begin{equation*}
v=\sqrt{\frac{\mu}{a}} \tag{14}
\end{equation*}
$$

Substituting Eq. (14) for $v$ in Eq. (13) yields

$$
\begin{equation*}
-\frac{d a}{2 \sqrt{\mu a} C_{b} \rho}=d t \tag{15}
\end{equation*}
$$

The time required for a spacecraft in a circular orbit to fall from an initial semi major axis $\left(a_{0}\right)$ to a final $a_{f}$ can be calculated by integrating Eq. (15)

$$
\begin{equation*}
\Delta t=\int_{a_{0}}^{a_{f}}-\frac{d a}{2 \sqrt{\mu a} C_{b} \rho} \tag{16}
\end{equation*}
$$

$C_{b}$ is a constant and can be factored out of the integral.

$$
\begin{equation*}
\Delta t C_{b}=\int_{a_{0}}^{a_{f}}-\frac{d a}{2 \sqrt{\mu a} \rho} \tag{17}
\end{equation*}
$$

If we assume that the density is a function of only altitude, then the density will also be a function of semi major axis for a circular orbit around a spherical Earth. With this assumption, $a$ is the only variable in the integral in Eq. (17). Thus, when evaluated, the solution to the integral will be a function of $a$ (call it $G(a)$ ). Eq. (17) becomes

$$
\begin{equation*}
\Delta t C_{b}=G\left(a_{f}\right)-G\left(a_{0}\right) \tag{18}
\end{equation*}
$$

This relation shows that the time required to drop from $a_{0}$ to $a_{f}$ varies linearly with the ballistic coefficient. If the time required for a satellite with $C_{b 1}$ to go from $a_{0}$ to $a_{f}$ is $\Delta t_{1}$, then the time ( $\left.\Delta t_{2}\right)$ required for a satellite with $C_{b 2}$ to achieve the same change in semi major axis can be written as

$$
\begin{equation*}
\Delta t_{2}=\frac{C_{b 1} \Delta t_{1}}{C_{b 2}} \tag{19}
\end{equation*}
$$

Additionally, for a circular orbit, mean motion is the time rate of change or true anomaly $(\theta)$ and is given by

$$
\begin{equation*}
n=\frac{d \theta}{d t} \tag{20}
\end{equation*}
$$

Multiplying Eq. (20) by Eq. (15) yields

$$
\begin{equation*}
\frac{d \theta}{d a}=-\frac{n}{2 \sqrt{\mu a} C_{b} \rho} \tag{21}
\end{equation*}
$$

Substituting Eq. (10) for $n$ in Eq. (21) yields

$$
\begin{equation*}
\frac{d \theta}{d a}=-\frac{1}{2 a^{2} C_{b} \rho} \tag{22}
\end{equation*}
$$

Multiplying both sides of Eq. (22) by $C_{b}(d a)$ (ballistic coefficient times differential change in semi major axis) and integrating yields

$$
\begin{equation*}
\Delta \theta C_{b}=\int_{a_{0}}^{a_{f}} \frac{d a}{2 a^{2} \rho} \tag{23}
\end{equation*}
$$

Once again, if density is a function of $a$, the integral in Eq. (23) will also be a function of $a$ when evaluated. If we call this function $P(a)$, the Eq. (23) becomes

$$
\begin{equation*}
\Delta \theta C_{b}=P\left(a_{f}\right)-P\left(a_{0}\right) \tag{24}
\end{equation*}
$$

This shows that the change in true anomaly as a satellite falls from $a_{0}$ to $a_{f}$ varies linearly with $C_{b}$. The average orbital angular velocity of the spacecraft from $a_{0}$ to $a_{f}$ can be calculated by dividing Eq. (24) by Eq. (18).

$$
\begin{equation*}
\omega_{a v g}=\frac{\Delta \theta}{\Delta t}=\frac{P\left(a_{f}\right)-P\left(a_{0}\right)}{G\left(a_{f}\right)-G\left(a_{0}\right)} \tag{25}
\end{equation*}
$$

Note that in Eq. (25), $C_{b}$ simplifies and $\omega_{\text {avg }}$ is a function of only the initial and final semi major axes. This proves that the average orbital angular velocity of a spacecraft from $a_{0}$ to $a_{f}$ is independent of spacecraft ballistic coefficient assuming that the orbit is circular and that density is a function of $a$ only. If the effects of zonal harmonics are considered, the orbital plane will begin to precess. The average rate of this precession due to the $J_{2}$ zonal harmonic is dependent on the orbit inclination and semi major axis and is given by ${ }^{18}$

$$
\begin{equation*}
\dot{\Omega}_{a v g}=-\left|\frac{3}{2} \frac{\sqrt{\mu} J_{2} R_{e}^{2}}{\left(1-e^{2}\right)^{2} a^{7 / 2}}\right| \cos (i) \tag{26}
\end{equation*}
$$

where $R_{e}$ is the Earth's radius and $J_{2}$ (the second zonal harmonic) is a constant that describes the oblateness of Earth. Given a numerically propagated trajectory, $\dot{\Omega}_{\text {avg }}$ can also be calculated by diving the total change in right ascension by the orbit lifetime. Over a given time interval, the change in right ascension can be approximated by

$$
\begin{equation*}
\Delta \Omega=\dot{\Omega}_{a v g} \Delta t \tag{27}
\end{equation*}
$$

Since $a$ is the only variable in the $\dot{\Omega}_{\text {avg }}$ equation as the spacecraft decays from $a_{0}$ to $a_{f}$, it can be shown that $\dot{\Omega}_{a v g}$ during this decay is independent of the spacecraft ballistic coefficient by applying the same logic utilized to develop Eq. (25).

## B. Limitations of This Procedure

In reality, even orbits that begin circular will not remain perfectly circular when under the influence of aerodynamic drag, even if there are no other perturbations. Both the altitude and velocity will oscillate slightly,

American Institute of Aeronautics and Astronautics
especially as the orbit decays and the drag force becomes stronger. This is because for any starting point in an initially circular orbit, the aerodynamic drag force will slightly reduce the instantaneous velocity at that point and make it the orbit apogee. The altitude at the orbit's new perigee will then be lower, the density higher, and the velocity greater than at the apogee, so the drag force will be greater. This will cause what was formerly the perigee to become the apogee. This cycle continues until de-orbit. Furthermore, the fact that Earth is more like an oblate spheroid than a perfect sphere results in the well-known $J_{2}$ gravitational perturbation and means that not only will the semi major axis oscillate, but the altitude will also vary for a given semi major axis depending on the current latitude. The rotation of the atmosphere with Earth also causes perturbations in the drag profile. The combination of these effects cause significant oscillations in velocity and altitude. Figure 3 illustrates this by displaying the velocity and altitude profiles for an orbit with initial mean orbital elements ( $a[\mathrm{~km}], e, \Omega[\mathrm{deg}], \omega[\mathrm{deg}], \theta[\mathrm{deg}], i[\mathrm{deg}])=(6978,0,180,0,225,45)$, aerodynamic drag with 1976 standard atmosphere and $C_{b}=.025$, and zonal harmonics through $J_{4}$.


Figure 3.Velocity and Altitude over Time during Decay Trajectory


Figure 4. JB2008 Mean Air Density vs. Altitude Based on Solar and Geomagnetic Activity

The oscillations shown in Figure 3 mean that the velocity, density, and drag profiles will be different than expected for a circular orbit and the use of Equations (19) and (25) will introduce errors that can accumulate over time. Additionally, the density is not, in reality, a function of only altitude (or semi major axis) and can vary widely at any given altitude due to solar and geomagnetic activity as seen in Figure $4{ }^{19}$.

All these factors introduce errors, but since the targeting algorithm is based on perturbations from a numerically propagated trajectory, the drag force fluctuations and deviations from the circular orbit will be captured in the numerical trajectory. Thus, the closer the new trajectory that is being analyzed is to the initial numerically propagated trajectory, the smaller the error in the analytical solution will be. As such, the relations in Eqs. (19), (24), and (25) are usable in the targeting algorithm which calculates the control parameters (ballistic coefficient profile) needed for the spacecraft to re-enter at a desired longitude and latitude.

## C. Analytically Calculating Re-Entry Location Based on Applied Controls

The targeting algorithm requires the ability to calculate where a satellite will de-orbit given a set of initial conditions and applied controls ( $C_{b 1}, C_{b 2}$, and $t_{s w a p}$ ). This can be performed by first propagating an initial trajectory and analyzing perturbations from this initial trajectory. Having an analytical mapping from the initial conditions and applied controls to the final impact location facilitates the rapid calculation and testing of numerous combinations of control parameters until the most desirable set of controls is found. This analytical mapping is defined based on a single numerically propagated reference trajectory as follows:

American Institute of Aeronautics and Astronautics

1. The initial trajectory is obtained by propagating from the initial conditions with a chosen $C_{b l o}$ until time $t_{s_{\text {old }}}$. After this time, the ballistic coefficient becomes $C_{b 20}$, and the trajectory is propagated until a specified final semi major axis is reached. This is considered the terminal point of the trajectory and occurs at time $t_{\text {term }}$. Below the terminal point, wide variations in the experienced drag force may occur and the circular orbit assumptions are no longer valid. After $t_{\text {term }}$, the satellite takes on a ballistic coefficient $C_{b_{-} \text {term }}$ and the trajectory is propagated until the re-entry point where the spacecraft is assumed to disintegrate due to aerodynamic heating. When generating new trajectories, it is assumed that the ballistic coefficient during the terminal phase of the new trajectory is the same as during the terminal phase of the initial trajectory. For this reason, the terminal phase can be characterized by an amount of time $\left(t_{\text {deorbit }}-t_{\text {term }}\right)$, a change in true anomaly $\left(\Delta \theta_{\text {term }}\right)$, and, if considering zonal harmonics, a change in right ascension $\left(\Delta \Omega_{\text {term }}\right)$ between the terminal point and the de-orbit point. For each new set of control parameters, the new location of the terminal point is analytically determined and $\Delta \theta_{\text {term }}$ and $\Delta \Omega_{\text {term }}$ are added to the true anomaly and right ascension at this point to estimate the new de-orbit point location. Because the environmental perturbations do not appreciably affect inclination and since any new trajectory will have the same ballistic coefficient profile after $t_{\text {term }}$ and hence approximately the same $\Delta \theta_{\text {term }}, \Delta \Omega_{\text {term }}$, and $\left(t_{\text {deorbit }}-t_{\text {term }}\right)$, the location of the terminal point required to target the desired re-entry latitude and longitude can be uniquely determined. The goal of the targeting algorithm is now to define a new trajectory that passes through the terminal point, as this will guarantee that the spacecraft re-enters at the desired latitude and longitude.
1.1. During propagation of the initial trajectory, the time, position, and velocity at each time step are recorded
1.2. The average orbital angular velocity of the propagated trajectory from $t_{0}$ to the current time $(t)$ and from $t$ to $t_{\text {term }}$ are also calculated and recorded at each time step
1.3. The average rate of change of right ascension over the course of the trajectory is also calculated and can be multiplied by a time interval to calculate the approximate change in right ascension over that time interval.
2. Equations (19), (25), and (27) are used to determine the terminal point location of a spacecraft with the same initial conditions and different $C_{b 1}, C_{b 2}$, and $t_{s w a p}$. This is done by breaking the new trajectory into three phases. Each phase is represented by an initial and final semi major axis ( $a_{i}$ and $a_{f}$ ) such that the spacecraft in the new trajectory and the initial trajectory do not change their ballistic coefficients between $a_{i}$ and $a_{f}$. This enables the analysis of the behavior (change in orbital elements over time) of the new trajectory in each phase based on the behavior of the reference trajectory in the corresponding phase ( $a_{i}$ to $a_{f}$ ).
2.1. Since the time and ballistic coefficient for each phase of the initial trajectory are known and the ballistic coefficient in the new trajectory in the corresponding phase is known, Eq. (19) can be utilized to calculate the time required to complete each phase of the new trajectory.
2.2. Eq. (25) leads to the assumption that the average angular velocity in each phase of the new trajectory is the same as the average angular velocity of the reference trajectory in the corresponding phase because both phases have the same $a_{i}$ and $a_{f}$. The average rates of change of right ascension of the old and new trajectories are also assumed to be the same in each phase.
2.3. Since the time and average angular velocity in each phase of the new trajectory are known, the total change in true anomaly in each phase can be found by multiplying the time by the average angular velocity.
2.4. The total change in right ascension during each phase can likewise be found by multiplying the average rate of change of right ascension by the time required for that phase.
2.5. The total orbit lifetime of the new trajectory until the terminal point can be found by adding the times required for each phase. Similarly, the total changes in true anomaly and right ascension until the terminal point can be found by summing the changes that occur during each phase.
2.6. Assuming that the true anomaly, right ascension, and semi major axis are the only orbital elements that are changing and $\Delta \theta_{\text {term }}, \Delta \Omega_{\text {term }}$, and $\left(t_{\text {deorbit }}-t_{\text {term }}\right)$ remain constant for any new trajectory, the time and orbital elements of the spacecraft at de-orbit can be calculated. These orbital elements can be converted to the ECI frame and can then be converted (using de-orbit time) to de-orbit latitude and longitude. Because we only care about de-orbit location, the orbital elements at the terminal point with $\Delta \theta_{\text {term }}$ added to the true anomaly, $\Delta \Omega_{\text {term }}$ added to the right ascension, and $\left(t_{\text {deorbit }}-t_{\text {term }}\right)$ added to the time can be used as the final orbital elements for the purpose of calculating final latitude and longitude.
2.7. A mapping now exists from the initial conditions and control parameters to the de-orbit location. Figure 5 illustrates the partitioning of the new and initial trajectories into phases bounded by the time points $t_{0}$, $t_{s_{-} n e w}$, $t_{s_{-} o l d}, t_{\text {eq_en }}, t_{\text {equiv }}, t_{f_{-} o l d}$, and $t_{f_{-} n e w}$ defined as follows:

- $\quad t_{0}$ : Initial time
- $\quad t_{s_{-} n e w}$ : New swap time
- $\quad t_{s_{-}}$old: Swap time in initial trajectory

American Institute of Aeronautics and Astronautics

- $\quad t_{e q_{-} e n}$ : Point in new trajectory where the energy of the orbit (and semi major axis) is the same as the energy of the initial trajectory at time $t_{s_{-}}$old
- $\quad t_{\text {equiv }}$ : Time in the initial trajectory at which the energy of the orbit is the same as the energy of the new trajectory at $t_{s_{-} n e w}$
- $\quad t_{\text {term_old }}$ : Time until the terminal point in the initial trajectory
- $\quad t_{\text {term_new }}$ : Time until the terminal point in the new trajectory

Note that this figure illustrates the case where $t_{\text {equiv }}$ is less than $t_{s_{\text {_o }}}$ old


Figure 5. Semi Major Axis over Time for Old and New Trajectories
Now that an analytical relationship has been developed relating the initial state and control parameters ( $C_{b 1}, C_{b 2}$, and $t_{\text {swap }}$ ) to the re-entry location, the effects of variations in the control parameters can be more easily analyzed. Note that while the trajectory phases are defined in terms of semi major axes, semi major axis values do not appear explicitly in the mapping from initial state to final state. Time values are utilized instead because even mean semi major axis measurements exhibit some oscillations while time progresses at a much more predictable rate.

## III. Latitude and Longitude Targeting Algorithm

Using the analytical relationship between the control parameters and de-orbit location developed in Section II, the tasks of latitude and longitude targeting can be decoupled, making it possible to analytically calculate the control parameters needed for re-entry point targeting. Latitude targeting will be addressed first. Assuming that $C_{b 1}$ and $C_{b 2}$ have different values and that maneuvering is initiated with sufficient time (quantified in Section IV) before de-orbit, it will be possible to target any latitude below the orbit inclination by varying only $t_{\text {swap }}$. To do this, a range of acceptable $t_{\text {swap }}$ values (discussed in Section IV) is first defined. From this range, the $t_{\text {swap }}$ value that yields the smallest correctable longitude error is chosen.

Using the relations discussed in Section II, the set of $t_{\text {swap }}$ values that yield perfect latitude targeting can be calculated semi-analytically. The first step is determining the angle $\phi$ at de-orbit where

$$
\begin{equation*}
\phi=\bmod (\omega+\theta, 2 \pi) \tag{28}
\end{equation*}
$$

is the angle between the ascending node and the current spacecraft position measured along the orbital track. Note that mod indicates the modulus operator. To do this, the z-component of the ECI position vector at the target latitude is first expressed in terms of the target latitude and the magnitude of the spacecraft position vector at the target latitude

$$
\begin{equation*}
R_{z}=r \sin (\text { lat })=\frac{h^{2}}{\mu(1+e \cos \theta)} \sin (\text { lat }) \tag{29}
\end{equation*}
$$

Define the perifocal coordinate system as having the origin at the orbit focus, the x -axis toward the periapsis, the z axis aligned with the angular momentum vector, and the $y$-axis completing the right handed coordinate system ${ }^{18}$. The conversion from the final orbital elements to a Cartesian position vector in the perifocal frame is calculated by

$$
\left[\begin{array}{l}
r_{x}  \tag{30}\\
r_{y} \\
r_{z}
\end{array}\right]=\frac{h^{2}}{\mu(1+e \cos \theta)}\left[\begin{array}{c}
\cos \theta \\
\sin \theta \\
0
\end{array}\right]
$$

The perifocal position vector can be multiplied by the direction cosine matrix given by Eq. 4.49 in Curtis' book ${ }^{18}$ to transform it into the ECI frame. The z-component of the ECI position vector can be solved for by multiplying only the bottom row of the perifocal to ECI direction cosine matrix by the perifocal position vector

$$
R_{z}=\frac{h^{2}}{\mu(1+e \cos \theta)}[\sin (\omega) \sin (i) \quad \cos (w) \sin (i) \quad \cos (i)]\left[\begin{array}{c}
\cos \theta  \tag{31}\\
\sin \theta \\
0
\end{array}\right]=\frac{h^{2}}{\mu(1+e \cos \theta)} \sin (l a t)
$$

Because our goal is to find the $\phi$ at which a spacecraft in a given keplarian orbit passes over the target latitude, we can assume, for the purposes of this calculation, that $\theta$ is the only variable changing over time and that the other orbital elements retain the same values over time. Strictly speaking, this assumption is not entirely accurate but the error it introduces if less than the other sources of error in the analytical solution since the changes in the other orbital elements over time are miniscule compared to the changes in $\theta$, and it transforms Eq. (31) into a function of one variable. Because $\theta$ must be between zero and $2 \pi$, the bisection root finding method can be utilized to rapidly calculate a value of $\theta$ that satisfies Eq. (31). From this, the corresponding $\phi$ can be readily calculated using Eq. (28). Now, there are always two values of $\phi$ for which the spacecraft is over the target latitude as illustrated in Figure 6. These two values are related by the equation


Figure 6. Angles from ascending node at target latitude

$$
\begin{equation*}
\bmod \left(\phi_{d 1}+\phi_{d 2}, 2 \pi\right)=\pi \tag{32}
\end{equation*}
$$

Regardless of which $\phi$ value the bisection method returns, the other can be calculated using Eq. (32).
For each calculated value of $\phi$, a value of $\phi+2 \pi n$ where $n$ is an integer will also provide proper latitude targeting. As will be shown in Section IV, the latitude controllability of the system can be assessed to determine the minimum and maximum values of $n$. For all possible $\phi$ values, the increase in true anomaly required for latitude targeting can be calculated by subtracting the initial angle $\phi_{i}$ from the desired $\phi_{d}$.

$$
\begin{equation*}
\Delta \theta_{d}=\bmod \left(\phi_{d}-\phi_{i}, 2 \pi\right) \tag{33}
\end{equation*}
$$

Section IV part A will discuss the analytical calculation of the $\Delta \theta_{d}$ that results from an increase in $t_{\text {swap }}$. The increase in $t_{\text {swap }}$ necessary to produce a desired $\Delta \theta_{d}$ is calculated by substituting Eq. (50) into Eq. (49) and solving for $\Delta t_{\text {swap }}$.

$$
\begin{equation*}
\Delta t_{s w a p}=\frac{\Delta \theta_{d} C_{b 2}}{\omega_{2_{\text {avg }}}\left(C_{b 2}-C_{b 1}\right)} \tag{34}
\end{equation*}
$$

where $\omega_{2_{\text {avg }}}$ is the average angular velocity during phase two of the orbit shown in Figure 7. The de-orbit locations associated with all $t_{\text {swap }}$ values that provide latitude targeting are recorded along with the corresponding longitude errors.

A positive longitude error (satellite east of impact point) means that the orbit would have to last longer with the same total change in true anomaly to achieve the desired longitude targeting. A negative longitude error (satellite west of impact point) means the orbit would have to last shorter to achieve the desired longitude targeting. The $t_{\text {swap }}$ value that yields the lowest correctable longitude error should be chosen. Determining the range of correctable longitude error is discussed further in Section IV (Controllability Analysis). Once the desired $t_{s w a p}$ value has been selected, the $C_{b 1}$ and $C_{b 2}$ values will be varied in such a way that the remaining longitude error is eliminated without causing additional latitude error. If the total change in true anomaly remains constant, the final impact latitude (and hence latitude error) will remain the same. The fact that the mean motion (average angular velocity) of the spacecraft at larger semi major axes is less than at lower semi major axes makes it possible to change the total orbit lifetime without varying the total change in true anomaly. An increase in orbit lifetime could be achieved by reducing $C_{b l}$ while increasing $C_{b 2}$. This would mean that the satellite spends more time at a greater semi major axis. Because this greater semi major axis means a slower mean motion, the satellite will orbit longer for the same total change in true anomaly. Conversely, to reduce the total orbit lifetime without varying the total change in true anomaly, $C_{b l}$ would be increased while $C_{b 2}$ would be reduced. This would mean that the satellite spends more time in the lower orbit and experiences a greater average mean motion. Thus, the satellite would experience a shorter orbit lifetime for a given total change in true anomaly.

American Institute of Aeronautics and Astronautics

From the definitions of the variables $\left(t_{s_{o l d}}, t_{s_{\text {new }}}, C_{b 10}, C_{b 20}, C_{b 1}, C_{b 2}, \Delta \theta_{10}, \Delta \theta_{20}, \Delta \theta_{1}, \Delta \theta_{2}, \Delta \theta_{t}\right.$, $\Delta t_{10}, \Delta t_{20}, \Delta t_{1}, \Delta t_{2}, \Delta t_{t}$ ) in the Nomenclature section and from Eqs. (19) and (24), we can quantify the effects of changes in the ballistic coefficients on impact location. Assuming also that the drag configurations are swapped at the same semi major axis in the new and initial trajectories:

$$
\begin{gather*}
\Delta \theta_{1}+\Delta \theta_{2}=\Delta \theta_{t}  \tag{35}\\
\Delta t_{1}+\Delta t_{2}=\Delta t_{t}  \tag{36}\\
\Delta \theta_{1}=\frac{\Delta \theta_{10} C_{b 10}}{C_{b 1}}  \tag{37}\\
\Delta \theta_{2}=\frac{\Delta \theta_{20} C_{b 20}}{C_{b 2}}  \tag{38}\\
\Delta t_{1}=\frac{\Delta t_{10} C_{b 10}}{C_{b 1}}  \tag{39}\\
\Delta t_{2}=\frac{\Delta t_{20} C_{b 20}}{C_{b 2}} \tag{40}
\end{gather*}
$$

We can now solve analytically for the $C_{b 1}$ and $C_{b 2}$ required (Eqs. (42) and (45)) to achieve the desired $\Delta \theta_{t}$ and $\Delta t_{t}$ (total change in true anomaly and total time required to reach the terminal point)

$$
\begin{gather*}
\Delta \theta_{t}=\Delta \theta_{1}+\Delta \theta_{2}=\frac{\Delta \theta_{10} C_{b 10}}{C_{b 1}}+\frac{\Delta \theta_{20} C_{b 20}}{C_{b 2}}  \tag{41}\\
C_{b 1}=\frac{\Delta \theta_{10} C_{b 10} C_{b 2}}{\Delta \theta_{t} C_{b 2}-\Delta \theta_{20} C_{b 20}}  \tag{42}\\
\Delta t_{t}=\frac{\Delta t_{10} C_{b 10}}{C_{b 1}}+\frac{\Delta t_{20} C_{b 20}}{C_{b 2}}  \tag{43}\\
\Delta t_{t}=\frac{\Delta t_{10}\left(C_{b 10}\right)\left(\Delta \theta_{t} C_{b 2}-\Delta \theta_{20} C_{b 20}\right)}{\Delta \theta_{10} C_{b 10} C_{b 2}}+\frac{\Delta t_{20} C_{b 20}}{C_{b 2}}  \tag{44}\\
C_{b 2}=\frac{C_{b 20}\left(\Delta t_{20} \Delta \theta_{10}-\Delta t_{10} \Delta \theta_{20}\right)}{\left(\Delta t_{t}\right)\left(\Delta \theta_{10}\right)-\left(\Delta t_{10}\right)\left(\Delta \theta_{t}\right)} \tag{45}
\end{gather*}
$$

In this case, $\Delta \theta_{t}$ will be the same as in the trajectory with $t_{\text {swap }}$ calculated for latitude targeting and $\Delta t_{t}$ will be the original orbit lifetime plus the desired increase in orbit lifetime necessary for longitude targeting $\left(\Delta t_{d}\right)$. Note that only the drag profile before the terminal point will be manipulated by the targeting algorithm, so the time to deorbit and total change in true anomaly of the new trajectory after the terminal point will be the same as for the initial trajectory after this point. Because Eqs. (35)-(45) assume that the swap points occur at the same semi major axes for the new and initial trajectories, it will be necessary to update $t_{s_{n e w}}$ so that this is the case. This is performed by imposing

$$
\begin{equation*}
t_{s_{\text {new }}}=\frac{t_{s_{o l d}} C_{b 10}}{C_{b 1}} \tag{46}
\end{equation*}
$$

This procedure finds a set of control parameters that will result in minimized latitude and longitude targeting errors. Once the procedure is completed, a new initial trajectory can be configured and propagated with the new $t_{s w a p}, C_{b l}$, and $C_{b 2}$ values. There may be some error between this newly propagated trajectory and the analytical solution due to the assumptions made in developing the relations used to calculate the analytical solution. However, the newly propagated trajectory will be closer to the ideal trajectory, the analytical solution process can be repeated, and the results can be used to configure and propagate yet another trajectory as illustrated in Figure 2. This trajectory will have a smaller deviation from the analytical solution. After a few iterations, the analytical solution should agree with the numerical propagation within some tolerance. At this point, the control parameters needed to reduce the latitude and longitude targeting errors below the specified tolerance will be calculated.

## IV. Controllability Analysis

## A. Latitude Controllability

Controllability is defined as the ability to achieve any desired final state in a finite amount of time from a given initial state and range of control parameters ${ }^{20}$. If not configured correctly, there may be some cases where the system is unable to target the desired longitude and latitude. This can happen if the maneuver is initiated with insufficient orbit life remaining, if poor initial $C_{b 1}, C_{b 2}$, and $t_{s w a p}$ values are chosen, or if the ballistic coefficient of the spacecraft cannot be varied significantly. This section investigates the factors that contribute to the controllability of the system and investigates the targeting capabilities of the system based on the initial state and available control parameters.


First, let us consider the effects on the impact location of deviations in the value of only $t_{\text {swap }}$ from an initial trajectory. Consider the case where $t_{s w a p}$ is increased while $C_{b 1}$ and $C_{b 2}$ remain constant. Changing $t_{\text {swap }}$ will mean that phase two of the new trajectory (the phase between $t_{s_{\text {old }}}$ and $t_{s_{\text {new }}}$ ) will have a different time and change in true anomaly than phase two of the initial trajectory (assuming that $C_{b 1}$ and $C_{b 2}$ are not identical). The total changes in true anomaly and times required for phases one and three of the new trajectory will be the same as in the initial trajectory as illustrated in Figure 7.

If $\Delta t_{20}$ is the time required for phase two of the initial trajectory, the time $\Delta t_{2}$ required for phase two in the new trajectory is calculated by Eq. (19) as

Figure 7. Effects of Only Changing Swap Time

$$
\begin{equation*}
\Delta t_{2}=\frac{C_{b 2} \Delta t_{20}}{C_{b 1}} \tag{47}
\end{equation*}
$$

This is valid because the initial trajectory has $C_{b 2}$ during phase two while the new trajectory has $C_{b 1}$ and both trajectories have the same $a_{0}$ and $a_{f}$ during this phase. The total increase in orbit lifetime resulting from the increase in $t_{\text {swap }}$ is given by

$$
\begin{equation*}
\Delta t_{d}=t_{2}-t_{20}=\Delta t_{\text {swap }}-t_{20} \tag{48}
\end{equation*}
$$

Using Eq. (47), Eq. (48) can be rewritten as

$$
\begin{equation*}
\Delta t_{d}=\Delta t_{\text {swap }}-\Delta t_{\text {swap }}\left(\frac{C_{b 1}}{C_{b 2}}\right)=\Delta t_{\text {swap }}\left(1-\frac{C_{b 1}}{C_{b 2}}\right) \tag{49}
\end{equation*}
$$

From this equation, we see that if $C_{b 1}$ is greater than $C_{b 2}$, the change in orbit lifetime will be negative given an increase in $t_{\text {swap }}$. This happens because the spacecraft would be spending more time with a higher ballistic coefficient if $t_{\text {swap }}$ is increased. The analysis for the case where $t_{\text {swap }}$ is decreased is similar, except that the change in $t_{s w a p}$ is equal to $-t_{20}$ instead of $t_{2}$, but Eq. (49) for $\Delta t_{d}$ also results. Once the change in orbit lifetime has been calculated, the difference in the total change in true anomaly between the new and old trajectories can be calculated by

$$
\begin{equation*}
\Delta \theta_{d}=\omega_{2_{-} a v g} \Delta t_{d} \tag{50}
\end{equation*}
$$

Where $\omega_{2_{-} a v g}$ is the average angular velocity during phase 2 and is calculated based on the initial trajectory. Eq. (50) is valid because all variations in orbit lifetime and changes in true anomaly occur during phase two. Also important is the ability to calculate the maximum possible $t_{\text {swap }}$ value. We know that if $t_{\text {swap }}$ in the new trajectory is set to its maximum possible value, the phase three in Figure 7 will not exist since the satellite will maintain $C_{b l}$ until the terminal point. Thus, the total orbit lifetime (and the new value of $t_{\text {swap }}$ ) will equal the old $t_{\text {swap }}$ value plus the new time during phase 2 . This can be written as

$$
\begin{equation*}
t_{s_{\max }}=t_{s_{o l d}}+\left(t_{\text {termold }}-t_{s_{o l d}}\right) \frac{C_{b 2}}{C_{b 1}} \tag{51}
\end{equation*}
$$

The $t_{\text {swap }}$ value selected by the latitude targeting algorithm must be less than the maximum $t_{\text {swap }}$ value. It is recommended that at least on the first targeting iteration, only $t_{\text {swap }}$ values near the middle of the feasible $t_{\text {swap }}$ range be candidates for selection, since longitude controllability is limited by $t_{s w a p}$ values very close to the beginning or end of the physically attainable range. Through numerous simulations, it was determined that selecting only $t_{\text {swap }}$ values between $25 \%$ and $75 \%$ of the maximum $t_{s w a p}$ was a reasonable constraint. Figure 8 and Figure 9 illustrate the increase in orbit lifetime and increase in the total change in true anomaly given a variation in $t_{\text {swap }}$. The initial conditions were
a 300 km circular orbit and the initial $t_{\text {swap }}$ was 150,000 seconds ( 41.67 hours) with ballistic coefficients in units of $m^{2} / k g$


Figure 8. Effects of Changes in Swap Time on Orbit Lifetime


Figure 9. Effects of Changes in Swap Time on Total Change in True Anomaly

The ability to change orbit lifetime by at least 12 hours ( 43,200 seconds) guarantees that the target longitude will pass beneath the orbital plane at least once and that the longitude error will be no greater than Earth's angle of rotation over half an orbital period. The worst case longitude error is thus about 1250 km for an equatorial target location. As illustrated by Eq. (49) and Eq. (50), changing $t_{\text {swap }}$ will have a greater effect on the total orbit lifetime and change in true anomaly if the ratio of $C_{b 1}$ to $C_{b 2}$ is greater. Thus,


Figure 10. The effects of changes in $\boldsymbol{t}_{\text {swap }}$ on total targeting for maximum controllability, it is best to propagate the initial trajectory with the largest ratio of $C_{b 1}$ to $C_{b 2}$ that the spacecraft is capable of achieving. In the current scenario, because the maximum variation in the total change in true anomaly is well over $2 \pi$ radians, there will be multiple $t_{\text {swap }}$ values that result in zero latitude error as shown Figure 10 for a 300 km circular orbit with ballistic coefficients in units of $m^{2} / \mathrm{kg}$. The rapid oscillations represent cycles where latitude error increases then decreases (eventually passing through zero) while some longitude error remains, while the slower oscillations represent increases and decreases in longitude error as the Earth rotates beneath the satellite's orbital plane.

From the set of possible swap times, one would want to pick the time that resulted in zero latitude error and the minimum correctable longitude error. Because there are multiple possible $t_{\text {swap }}$ values to choose from, longitude error can often be made quite small (a few hundred kilometers or less) through only a variation of $t_{\text {swap }}$.

## B. Longitude Controllability

Once the most desirable $t_{s w a p}$ value has been determined, $C_{b 1}$ and $C_{b 2}$ must be varied to eliminate the remaining longitude error by changing the orbit lifetime without varying the total change in true anomaly. The maximum amount by which orbit lifetime can be varied will depend on the characteristics of the initial trajectory and the selected $t_{\text {swap }}$ value. Recognizing that

$$
\begin{gather*}
\Delta \theta_{10}=\omega_{10} \Delta t_{10}  \tag{52}\\
\Delta \theta_{20}=\omega_{20} \Delta t_{20}  \tag{53}\\
\Delta \theta_{t}=\Delta \theta_{10}+\Delta \theta_{20}+\Delta \theta_{d}  \tag{54}\\
\Delta t_{t}=\Delta t_{10}+\Delta t_{20}+\Delta t_{d} \tag{55}
\end{gather*}
$$

We can rewrite Eq. (45) as

$$
\begin{equation*}
C_{b 2}=\frac{C_{b 20}\left(\Delta t_{20}\left(\omega_{10}-\omega_{20}\right)\right)}{\left(\Delta t_{20}\right)\left(\omega_{10}-\omega_{20}\right)+\left(\Delta t_{d}\right) \omega_{10}-\left(\Delta \theta_{d}\right)} \tag{56}
\end{equation*}
$$

American Institute of Aeronautics and Astronautics

Assuming $\Delta \theta_{d}=0$ (no desired difference in change in true anomaly between the trajectories) and solving for $\Delta t_{d}$ yields

$$
\begin{equation*}
\Delta t_{d}=\frac{\Delta t_{20}\left(\omega_{10}-\omega_{20}\right)}{\omega_{10}}\left(\frac{C_{b 20}}{C_{b 2}}-1\right) \tag{57}
\end{equation*}
$$

For a given value of $C_{b 2}$, the $C_{b 1}$ needed to ensure $\Delta \theta_{d}=0$ is calculated using Eq. (42) and the resulting increase in orbit lifetime $\left(\Delta t_{d}\right)$ is given by Eq. (57). Because $\omega_{10}<\omega_{20}$ (since $\omega_{20}$ applies to a lower orbit), $\omega_{10}-\omega_{20}<0$. Thus, if $C_{b 2}>C_{b 20}$, then $\Delta t_{d}>0$ and if $C_{b 2}<C_{b 20}$, then $\Delta t_{d}<0$. Therefore, in order to increase $\Delta t_{d}, C_{b 2}$ should be increased. This will result in a decreased value of $C_{b 1}$ required to maintain the same total change in true anomaly. To be able to achieve the maximum possible increase in orbit lifetime, the initial trajectory should be propagated with the lowest possible $C_{b 2}$ value and the highest possible $C_{b 1}$ value. This will also increase the effects of variations in $t_{s w a p}$ on the system which will mean that the longitude error can be reduced just through changing $t_{\text {swap }}$. Positive longitude errors mean that the spacecraft has landed East of the target point and are remedied by an increase in orbit lifetime while negative longitude errors are remedied by a decrease in lifetime. Figure 11 illustrates the $C_{b}$ values required to achieve various increases in total orbit lifetime for a 300 km initial circular orbit with a $t_{\text {swap }}$ value of 150,000 seconds (41.67 hours). Note that for certain $\Delta t_{d}$ values, the required $C_{b 1}$, and $C_{b 2}$ values may not be physically attainable.


Figure 11. $C_{b}$ Values Required to Produce Given Changes in Orbit Lifetime ( $\boldsymbol{C}_{b 10}=.0515, \boldsymbol{C}_{b 20}=.01717$ )


Figure 12. Maximum Possible Change in Orbit Lifetime for Various Swap Times

The maximum and minimum $\Delta t_{d}$ values (for $\Delta \theta_{d}=0$ ) are determined by the minimum and maximum $C_{b 1}$ and $C_{b 2}$ values. To find the maximum $\Delta t_{d}$ value, choose the maximum possible $C_{b 2}$ value that does not require $C_{b 1}$ to be below the minimum value. To do this, first substitute the maximum $C_{b 2}$ value into Eq. (42) to calculate the $C_{b 1}$ required to keep $\Delta \theta_{d}=0$ for the given $C_{b 2}$.

$$
\begin{equation*}
C_{b 1}=\frac{\Delta \theta_{10} C_{b 10} C_{b 2}}{\left(\Delta \theta_{10}+\Delta \theta_{20}\right) C_{b 2}-\Delta \theta_{20} C_{b 20}} \tag{58}
\end{equation*}
$$

If the required $C_{b l}$ is above its minimum value, then the limiting factor is the maximum value of $C_{b 2}$. The maximum value of $\Delta t_{d}$ can thus be calculated by substituting the maximum $C_{b 2}$ into Eq. (57). If $C_{b 1}$ is below the minimum possible value, then the limiting factor is $C_{b 1}$. The $C_{b 2}$ required to maintain $\Delta \theta_{d}=0$ for a given value of $C_{b 1}$ is calculated in a similar manner to Eq. (42) as

$$
\begin{equation*}
C_{b 2}=\frac{\Delta \theta_{20} C_{b 20} C_{b 1}}{\left(\Delta \theta_{10}+\Delta \theta_{20}\right) C_{b 1}-\Delta \theta_{10} C_{b 10}} \tag{59}
\end{equation*}
$$

If $C_{b 1}$ is the limiting factor, substituting the minimum value of $C_{b 1}$ into Eq. (59) provides the value of $C_{b 2}$ that yields the maximum $\Delta t_{d}$ without exceeding the range of feasible spacecraft ballistic coefficients. This value of $\Delta t_{d}$ can be found by substituting the newly calculated $C_{b 2}$ into Eq. (57). To achieve the minimum value of $\Delta t_{d}$ for $\Delta \theta_{d}=0$, the minimum possible $C_{b 2}$ and the maximum possible $C_{b 1}$ will be desired. Eq. (58) is solved using the minimum $C_{b 2}$ and if the required $C_{b l}$ is above the maximum value, $C_{b 1}$ is the limiting factor, and the required $C_{b 2}$ can be calculated by substituting the maximum value of $C_{b l}$ into Eq. (59). As before, the minimum $\Delta t_{d}$ is found by solving Eq. (57) with the calculated $C_{b 2}$ value. Note that the minimum $\Delta t_{d}$ is generally negative. Before performing longitude targeting, it is a good idea to calculate the minimum and maximum values of $\Delta t_{d}$ to ensure that only feasible combinations of $C_{b l}$ and $C_{b 2}$ are returned. If the desired $\Delta t_{d}$ is out of the feasible range, the system should only try to correct the longitude error to the extent possible within the controllability limits of the spacecraft. During the latitude targeting phase, it is
best to pick a $t_{\text {swap }}$ that results in a longitude error that is correctable. It is important to note that the closer a $t_{\text {swap }}$ value is to the beginning or end of the orbit lifetime, the more limited the longitude controllability will be. Figure 12 illustrates this by displaying the maximum $\Delta t_{d}$ for various $t_{\text {swap }}$ values for a 300 km initial circular orbit with $C_{b 10}=$ .0515 and $C_{b 20}=.01717$ and maximum and minimum $C_{b}$ values of .1 and $.01 \mathrm{~m}^{2} / \mathrm{kg}$ respectively. For this particular set of initial conditions, it is clear to see that there is sufficient controllability to target nearly any desired de-orbit location with a latitude below the orbit inclination.

## C. Sensitivity Analysis

Once the control parameters ( $C_{b 1}, C_{b 2}$, and $t_{s w a p}$ ) necessary for latitude and longitude targeting have been determined, it is useful to understand the effects of deviations from these control parameters on the final impact location. In reality, the control parameters represent a drag profile necessary to de-orbit in the desired location. For example, a two percent increase in the ambient density will have the same effect as a 2 percent increase in the ballistic coefficient. We will first investigate the case of a 300 km circular orbit around a spherical Earth with the 1976 standard atmosphere with initial control parameters $C_{b 10}=.01 \frac{\mathrm{~m}^{2}}{\mathrm{~kg}}, C_{b 20}=.025 \frac{\mathrm{~m}^{2}}{\mathrm{~kg}}, t_{s 0}=150,000 \mathrm{~s}$. Figure 13 illustrates the effects of variations in $C_{b 2}$ on the de-orbit location. As Figure 13 shows, the system is very sensitive to differences between the actual and expected drag forces. A total error of one fourth of one percent in the drag force estimate may cause the spacecraft to land on the other side of the Earth. If the simulator were a perfect representation of reality, the ballistic coefficient profile could be applied open loop and the spacecraft would follow the guidance trajectory and de-orbit in the required location. In reality, errors in the drag estimate will arise due to difficulties in forecasting the density and estimating the drag coefficient. To compensate for this, a feedback control system must be utilized to continuously vary the ballistic coefficient to ensure that the spacecraft experiences the drag profile required to follow the desired decay trajectory.

## V. Environmental Force Modeling



Figure 13. Effects of Changes in $\boldsymbol{C}_{\boldsymbol{b} 2}$ on De-Orbit Location

The goal of the targeting algorithm discussed in this paper is to generate a guidance (desired trajectory) that the spacecraft must follow to de-orbit at the desired point. This guidance trajectory does not have to be a perfect representation of how the spacecraft will behave, but it should be close enough to reality that any errors in the force model can be corrected by modulating the ballistic coefficient to keep the spacecraft on the guidance. Because changes in the ballistic coefficient only affect aerodynamic drag, only errors in the prediction of forces in the along-track direction can be directly corrected. Drift from the guidance trajectory in the radial or out of plane directions can only be remedied by a re-computation of the guidance trajectory. Fortunately, out of plane perturbations (namely $J_{2}$ ) are well known and can be modelled accurately enough that the need for guidance recomputation due to radial or out of plane drift will be rare. By far the most difficult to model environmental force is aerodynamic drag due to uncertainties in the drag coefficient and density. Fortunately, errors in the drag force estimate can be corrected through variation of the spacecraft's ballistic coefficient. This section will discuss the modeling of environmental forces and methods for incorporating them into the targeting algorithm.

## A. Gravitational Force and Perturbations

Gravitational perturbations are divided into zonal harmonics which capture variations in Earth's gravity at different latitudes, sectorial harmonics which capture longitude-dependent gravitational effects, and tesseral harmonics which capture gravitational effects that are dependent on both longitude and latitude ${ }^{21}$. Gravitational models such as EGM2008 ${ }^{22}$ provide the normalized gravitational coefficients ( $C$ and $S$ ) that are utilized to calculate the gravitational potential function at a given location using Equation 8-21 in Vallado's book ${ }^{17}$

$$
\begin{equation*}
U=\frac{\mu}{r}\left[1-\sum_{l=2}^{\infty} J_{l}\left(\frac{R_{e}}{r}\right)^{l} P_{l}\left[\sin \left(\phi_{g c}\right)\right]+\sum_{l=2}^{\infty} \sum_{m=1}^{l}\left(\frac{R_{e}}{r}\right)^{l} P_{l, m}\left[\sin \left(\phi_{g c}\right)\right]\left\{C_{l, m} \cos (m \lambda)+S_{l, m} \sin (m \lambda)\right\}\right] \tag{60}
\end{equation*}
$$

Where $\lambda$ and $\phi_{g c}$ are longitude and geocentric latitude and

American Institute of Aeronautics and Astronautics

$$
\begin{gather*}
J_{l}=-C_{l, 0}  \tag{61}\\
P_{l}[\gamma]=\frac{1}{2^{l} l!} \frac{d^{l}\left(\gamma^{2}-1\right)^{l}}{d \gamma^{l}}  \tag{62}\\
P_{l, m}[\gamma]=\left(1-\gamma^{2}\right)^{m / 2} \frac{d^{m}}{d \gamma^{m}} P_{l}[\gamma] \tag{63}
\end{gather*}
$$

The first summation in Eq. (60) represents the gravitational perturbations due to the zonal harmonics while the second represents the perturbations due to the tesseral and sectorial harmonics. Note that Eq. 8-22 in Vallado's book ${ }^{17}$ must be utilized to convert the normalized coefficients found in the EGM2008 table to the un-normalized coefficients needed for Eq. (60). The acceleration due to gravity at a given point is the gradient of Eq. (60). Beyond $J_{2}$, the zonal, spherical, and tesseral harmonics perturb the orbit by roughly the same order of magnitude as evidenced by the similarity of the un-normalized gravitational coefficients. At minimum, a simulation must include the $J_{2}$ effects, but the simulator used for this work included the effects of the $J_{3}$ and $J_{4}$ zonal harmonics as well because they could be added with little additional computation requirements. Page 421 of the book by Bate, Muller, and White ${ }^{23}$ gives a closed form solution for the gravitational acceleration in the ECI frame at a given position based on the Zonal harmonics through $J_{6}$. Only the ECI acceleration due to $J_{2}$ and two-body gravity will be repeated here for brevity

$$
\begin{gathered}
\ddot{x}=-\frac{\mu x}{r^{3}}\left[1-\frac{3}{2} J_{2}\left(\frac{R_{e}}{r}\right)^{2}\left(\frac{5 z^{2}}{r^{2}}-1\right)\right] \\
\ddot{y}=\frac{y}{x} \ddot{x} \\
\ddot{z}=-\frac{\mu z}{r^{3}}\left[1+\frac{3}{2} J_{2}\left(\frac{R_{e}}{r}\right)^{2}\left(3-\frac{5 z^{2}}{r^{2}}\right)\right]
\end{gathered}
$$

For higher fidelity simulations, the derivative of Eq. (60) or open source software packages such as GeographicLib ${ }^{24}$ can be utilized to calculate the effects of the higher order gravitational coefficients.

If $J_{2}$ effects (and possibly higher order) are included in the gravitational model and the calculation of orbital elements is required, it is best to use mean orbital elements instead of the traditional osculating elements. The mean elements serve to average out the short periodic oscillations in the osculating orbital elements caused by the $J_{2}$ gravitational perturbations. Some mean orbital element sets account for the periodic effects of higher order gravitational perturbations, but these yield limited accuracy improvements at the expense of simplicity, so only $J_{2}$ mean element sets will be considered here. Each osculating orbital element can be written as the corresponding mean orbital element plus the short term periodic variation due to $J_{2}$. The short term periodic variations of each element are given in terms of the current mean elements on pages 653-654 of Vallado's book ${ }^{17}$. For brevity and since semi-major axis is the most important mean orbital element in the targeting algorithm, only the conversion from mean to osculating semi major axis will be listed here. Given a set of mean elements, the osculating semi major axis is calculated by

$$
\begin{align*}
& a_{\text {osc }}=a_{\text {mean }}+\Delta a_{s p}  \tag{65}\\
& \Delta a_{s p}=\frac{J_{2} R_{e}^{2}}{a}\left[\left(\frac{a}{r}\right)^{3}-\frac{1}{\left(1-e^{2}\right)^{\frac{3}{2}}}+\left\{-\left(\frac{a}{r}\right)^{3}+\frac{1}{\left(1-e^{2}\right)^{\frac{3}{2}}}+\left(\frac{a}{r}\right)^{3} \cos (2 \omega+2 v)\right\} \frac{3 \sin ^{2}(i)}{2}\right] \tag{66}
\end{align*}
$$



Figure 14. Mean and Osculating Semi Major Axes for a 45 Degree Inclined Circular Orbit

To convert from osculating to mean orbital elements, the original osculating elements can be used to estimate the short term period variations which can then be utilized to estimate the mean elements. These mean element estimates will have some small errors since the variations are functions of mean elements, not osculating elements. They can, however, be utilized to compute more accurate estimates of the short term period variations. These can once again be used to estimate the mean orbital elements and the iterations can continue until accurate mean elements are calculated. Convergence usually occurs after less than 5 iterations. In the targeting algorithm, mean orbital elements are used to specify spacecraft initial conditions, to determine the terminal point for the first targeting iteration (based on mean semi major axis), and to display graphical state data to the user. Figure 14 shows the mean and osculating semi major axis of an orbit with an

American Institute of Aeronautics and Astronautics
initial mean semi major axis of 6698 km , an inclination of 45 degrees, and no aerodynamic drag. If an osculating semi major axis of 6998 km were used as an initial condition, it would be unclear whether the peak or trough of the semi major axis oscillation would occur at 6998 km .

## B. Aerodynamic Drag

Aerodynamic drag force is discussed in Vallado's book and is calculated by ${ }^{17}$

$$
\begin{equation*}
F_{d}=\frac{1}{2} C_{d} A \rho v_{r e l}^{2} \tag{67}
\end{equation*}
$$

Drag is by far the most difficult force to predict due to uncertainties in $C_{d}$ and $\rho$. Assuming a completely specular reflection of particles whereby each particle collides elastically with the satellite, the drag coefficient has a lower bound of 2 for a sphere and an upper bound of 4 for a flat plate ${ }^{10}$. The particles in low earth orbit, however, do not exhibit completely specular reflection and ionization of the particles due to Earth's magnetic field also has an effect on the drag coefficient. Sophisticated models based on theory and actual satellite observations have been developed to more accurately calculate the drag coefficient ${ }^{25}$, but these methods are not the focus of this paper. Additionally, the ballistic coefficient which contains the drag coefficient and is given by Eq. (12) can be estimated based on the observed orbital decay of the satellite. The minimum and maximum ballistic coefficient values are provided as arguments to the targeting algorithm and they are utilized to generate the guidance trajectory.

Density is also a highly uncertain parameter in the drag force equation. Because density can vary by up to two orders of magnitude at a given altitude based on time of day, latitude, longitude, and solar and geomagnetic activity as shown in Figure 4, an atmospheric model such as the 1976 standard atmosphere that provides density as a function of altitude is simply not sufficient for the generation of an accurate guidance. For guidance generation, errors between drag force estimates and the actual drag force must be correctable through modulation of the spacecraft's ballistic coefficient. By generating the guidance assuming a smaller range of feasible spacecraft $C_{b}$ values than is actually the case, the spacecraft can utilize the excess controllability margin to correct for errors in the drag force estimate. Since the atmospheric density around Earth at any given altitude can vary significantly at any point in time as shown in Figure 15, it is necessary to use models such as DTM, NRLMSISE-00, JB2008, or GOST ${ }^{17}$ that calculate the density at a given location using a combination of solar and geomagnetic activity data, historic satellite data, and atmospheric theories. The intricacies of these models are beyond the scope of this paper but are discussed in greater detail in section 8.6.2 of Vallado's book ${ }^{17}$.



Figure 15. MSISE Density around the Earth at a 300 km Altitude
The NRLMSISE-00 model was utilized in this work because it was one of the most modern and high performing ${ }^{17}$ and an implementation was readily available in the MATLAB aerospace toolbox. In addition to latitude, longitude, altitude, and time, the NRLMSISE-00 model takes as inputs the F10.7 solar indices and the Ap geomagnetic indices. Details about the inputs and implementation of MATLAB's atmosnrlmsise 00 function are provided on the MathWorks Website ${ }^{26}$. Historic F10.7 and Ap data can be found online at NASA's OmniWeb site ${ }^{27}$. For the purposes of this work, a table of historic F10.7 and Ap values was created using the OmniWeb data and referenced every time the NRLMSISE-00 model was called. For measurement intervals when the solar and geomagnetic data was missing, estimated values were created via a linear interpolation between the closest available data points. In practice, F10.7 and Ap forecasts should be uplinked to the satellite and used for the guidance generation algorithm. 45 day forecasts
of F10.7 and Ap are available online from the National Oceanic and Atmospheric Administration (NOAA) ${ }^{28}$. Marcos Et. $\mathrm{Al}^{29}$ discuss the accuracy of various density models by comparison with satellite data. Based on 69,932 density measurements on satellite between 200 and 620 km altitudes, the NRLMSISE-00 model exhibited a mean ratio of measured to actual density of $99.49 \%$ with a standard deviation of $17.17 \%$. The distribution was Gaussian, so this means that $68.3 \%$ of all density estimated were within $17.17 \%$ of the mean. The standard deviation was even less at lower altitudes with values of around $16 \%$ and $12 \%$ at 400 and 300 km respectively. Note that these errors assume accurate F10.7 and Ap values. If using forecasts, the F10.7 and Ap values themselves will also have errors ${ }^{25}$ which will cause additional inaccuracies in the density estimate. Still, the use of forecasted indices and a high fidelity density model should provide sufficient accuracy such that errors in the predicted drag force can be corrected.

While a high fidelity density model can be utilized in all iterations of the targeting algorithm, this presents two potential problems. The first is that the simulations take significantly longer to run and the second is that between iterations, the spacecraft trajectory may change slightly and different regions of density may be encountered. This may cause the density vs. altitude profile to vary by a small amount. Due to the sensitivity of the system to drag changes (see Figure 13), even a fraction of a percent change in density may result in a targeting error of thousands of kilometers. Failure to reduce the targeting error between iterations may lead to premature termination of the targeting algorithm. To remedy this, a "New Standard Atmosphere" can be created based on a propagation using the NRLMSISE-00 density model. This is done by running one latitude targeting sequence using NRLMSISE-00 density to get the spacecraft as close to the desired re-entry point as possible through a variation of only $t_{\text {swap }}$. The density, time, and position at each integration time step are recorded for this closest trajectory. The scale height $(H)$ at each point can also be calculated using the approach discussed on page 566 of Vallado's book ${ }^{17}$.

$$
\begin{equation*}
H=\frac{k T}{m_{w} g} \tag{68}
\end{equation*}
$$

where $k$ is Boltzmann's Constant, $T$ is temperature, $m_{w}$ is the average weight of each molecule ( $\mathrm{kg} / \mathrm{molecule}$ ), and $g$ is the acceleration due to gravity. The NRLMSISE-00 model provides the mass density of helium, oxygen $\left(\mathrm{O}\right.$ and $\left.\mathrm{O}_{2}\right)$, nitrogen ( N and $\mathrm{N}_{2}$ ), argon and hydrogen at any given point. The knowledge of the molecular weight (from the periodic table) and mass density of each gas can be utilized with the method of weighted averages to calculate the total average molecular weight of the air. Temperature is also provided by the NRLMSISE- 00 model and gravity can be calculated based on the current position in orbit. Ultimately, a table of altitude, density, and scale height at each point during the propagation can be calculated and sorted in ascending order by altitude.

Once this is done, the atmosphere can be divided into altitude bands and the density in each altitude band can be approximated using an exponential density model of the form given in Eq. 8-33 in Vallado's book ${ }^{17}$.

$$
\begin{equation*}
\rho=\rho_{0} e^{\left(-\frac{h-h_{0}}{H}\right)} \tag{69}
\end{equation*}
$$

Where $h$ is the current altitude, $h_{0}$ is the base altitude of the band, $H$ is the scale height through the band, and $\rho_{0}$ is the density at $h_{0}$. The value of $\rho_{0}$ in each band can be estimated based on the recorded density points in that band. Once all bands have been created, the scale height of each band can be adjusted such that the calculated density at the highest altitude in the band equals the base density of the next band. This ensures a continuous density vs. altitude function. The altitude range in each band is a parameter that can be set by the user with smaller ranges providing better density approximations and wider ranges providing a smoother density vs. altitude curve. Through repeated trials, an altitude range of 5 km per band was found to be a good balance. For regions such as the latter end of the orbit where there were very few recorded points, the altitude range per band was allowed to expand until each band had at least 100 measurement points up to a maximum altitude range of 15 km per band. Figure 16 displays a scatter plot of densityaltitude pairs encountered while propagating a decay trajectory using the NRLMSISE-00 model and the average density vs. altitude profile created using these points and the procedure discussed above. While the use of this "New Standard Atmosphere" for future propagations introduces some small density estimation errors, it significantly reduces the computation time required to propagate an orbit and significantly reduces the number of targeting iterations required for convergence since the density vs. altitude profile remains the same for each run. Figure 17 illustrates the percent error between the orbit averaged NRLMSISE-00 density and the density estimated by the new exponential atmospheric model. Figure 17 represents a nearly worst case density variation as a trajectory was chosen during a period of high density variability. Though this density error would likely be correctable through modulations of the spacecraft's ballistic coefficient, it may be possible to further reduce the error by tweaking the altitude bands or generating multiple new exponential atmospheres, each corresponding to a particular time span. These potential improvements will be investigated in future work.


Figure 16. NRLMSISE-00 and Average Density vs. Altitude for Circular Orbit with i=45 degrees, Epoch = [2004551200]

The rotation of the atmosphere also significantly affects the aerodynamic drag experienced by the satellite. The atmosphere tends to rotate with Earth due to viscous forces and has an average rotation rate between .7 and 1.4 times Earth's rotation rate $\left(\omega_{e}\right)$ based on altitude and latitude ${ }^{30}$. The rotation rate between 200 and 320 km (the range in which targeting usually takes place) is generally between 1 and $1.2^{30}$. As such, it is reasonable to assume that the atmospheric rotation rate is $\omega_{e}$. Taking this into account, the velocity vector of the satellite relative to the rotating atmosphere is ${ }^{17}$

$$
\begin{equation*}
\vec{v}_{r e l}=\frac{d \vec{r}}{d t}-\vec{\omega}_{e} \times \vec{r} \tag{70}
\end{equation*}
$$

This velocity should be used in the aerodynamic drag equation (Eq. (67)) for maximum accuracy. Atmospheric winds can also be taken into account in the calculation of $v_{\text {rel }}$, but wind data is seldom available and the effects are small enough to be ignored. Wind velocity is on the order of $100 \mathrm{~m} / \mathrm{s}$ at an altitude of $300 \mathrm{~km}^{31}$ which translates to a potential variation in drag force of up to six percent. This error in estimated drag force can be corrected easily through modulation of the spacecraft's ballistic coefficient.

## C. Other Perturbations

Other environmental perturbations include solar radiation pressure, solar gravity, and lunar gravity. For spacecraft in low Earth orbits, these effects are quite small and are generally less significant than even the $J_{5}$ gravitational perturbation. Though they can be included, it is generally not worth the effort or the increase in computational requirements to include them. This is because the goal of the guidance trajectory is not to be an absolutely perfect estimate of how the spacecraft will behave but to be an estimate with errors small enough to be correctable via changes in the spacecraft's ballistic coefficient and periodic re-computation of the guidance.

## VI. Simulation Implementation and Results

## A. Targeting Algorithm Process

The targeting algorithm uses the previously discussed principles and procedures to generate a ballistic coefficient profile that a spacecraft must follow to de-orbit in a desired location. The trajectory associated with this ballistic coefficient profile is called the guidance and the spacecraft will continually modulate its ballistic coefficient to track this guidance. This algorithm is designed to be implemented onboard a spacecraft and the current software implementation follows the flow chart in Figure 18.


Figure 18. Targeting Algorithm Flow Chart
In this implementation, the terminal point was set to a mean semi major axis of 6598 km and the spacecraft was considered to have re-entered the atmosphere when its distance from the center of Earth was 6498 km (approximately 120 km altitude). For propagations beyond the first one, the terminal point was specified by a time value ( $t_{\text {term }}$ ) instead of by a mean semi major axis value since even mean semi major axis oscillates slightly while time moves at a practically constant rate. This was done by estimating the times required during phases one, two, and three of each new trajectory (see Figure 5) using the procedure discussed in Section II part C and summing the times together to get the new $t_{\text {term }}$. The passage of time $t_{\text {term }}$ was set as a stopping condition for the propagator during phase three. This reduces error by avoiding any explicit calculation of semi major axis. In the last latitude targeting iteration, the system tries to minimize total longitude error without regard to longitude controllability.

This method provided targeting convergence even for highly perturbed, eccentric orbits because the effects of the perturbations and noncircular orbits were captured in the numerically propagated trajectories. Since the analytical solutions were based on these numerically propagated trajectories, the error was minimized.

## B. Monte Carlo Simulations

To analyze the effectiveness of the targeting algorithm, a set of 1000 Monte Carlo simulations with various initial conditions and targeting locations was conducted. Parameters were randomly selected from a uniform distribution within the ranges given in Table 1.
Table 1. Monte Carlo Simulation Parameters

| Variable | Range | Distribution |
| :---: | :---: | :---: |
| Semi Major Axis | $[6668,6778] \mathrm{km}$ | Uniform |
| True Anomaly | $[0,360]$ degrees | Uniform |
| Eccentricity | $[0, .004]$ | Uniform |
| Right Ascension | $[0,360]$ degrees | Uniform |
| Argument of the Periapsis | $[0,360]$ degrees | Uniform |
| Inclination | $[1,97]$ degrees | Uniform |
| Impact Latitude | $[0$, inclination-.001] degrees | Uniform |
| Impact Longitude | $[-180,180]$ degrees | Uniform |
| $\mathrm{Cb}_{\max }$ | $[.033, .067]$ | Uniform |
| $\mathrm{Cb}_{\min }$ | $[.0053, .027]$ | Uniform |
| Epoch | $[11 / 1 / 2003,11 / 1 / 2014]$ | Uniform |

American Institute of Aeronautics and Astronautics

The semi major axis range corresponds to average altitudes between approximately 290 and 400 km . This is a common range for deployment of LEO satellites, and targeting would be unlikely to begin at any higher altitude. Note that while orbits at the upper altitude range may last for several months, targeting will not begin until the orbit has decayed such that the specified level of longitude controllability is available. The upper bound on eccentricity is set to . 004 because the mean eccentricity of the International Space Station has not exceeded .004 based on data between the years 2000 and 2016 from the STK spacecraft database ${ }^{16}$. Satellites in low Earth orbits tend to naturally circularize due to higher drag at the perigee while the space station experiences greater eccentricities to due to the frequent thrusting maneuvers required to maintain altitude. The space station's eccentricity thus serves as a reasonable upper bound for the eccentricity a non-thrusting satellite would experience at the time targeting began. The inclination range represents the inclination of most existing satellites and it is unlikely that any satellite would be launched outside of this range. The desired impact longitude is unbounded and the latitude is constrained to being below the orbit inclination. The minimum and maximum ballistic coefficient ranges correspond to what may be reasonable for small, low Earth orbit satellite with retractable drag devices. The Epoch range spans 11 years because average density experiences a long term cyclic variation with a period of 11 years corresponding to the solar cycle. The initial true anomaly, right ascension, and argument of the periapsis are unconstrained. Runs were conducted on a desktop PC with a 3.6 Ghz Intel i7 processor using a MATLAB R2016a orbit propagator with force models based on the theory from Section V. After 1000 Monte Carlo simulation runs, all cases had an error below 1000 km save one with an error of 1171 km . Figure 19 shows the distribution of latitude and longitude errors for all of the runs.


Figure 19. Latitude and Longitude Errors (right graph zoomed in)
Table 2 displays the averages of important simulation results. Figure 20 displays a histogram of the targeting errors and the error cumulative distribution function (cdf) which gives the probability of achieving an error below a specified amount. To get the cdf, the MATLAB fitdist function was first called with the 'kernel' argument. This function estimates the probability density function (pdf) of the given data using a non-parametric kernel distribution. This continuous pdf can then be integrated to get the cdf. Note that the integral of the pdf from zero to $x$ represents the probability of the error of a given run falling below $x$ kilometers.
Table 2. Average Simulation Results

| Total Error (km) | Longitude Error (km) | Latitude Error (km) | Orbit Lifetime (days) | Sim. Run Time (mins) |
| :---: | :---: | :---: | :---: | :---: |
| 61.8 | 53.4 | 16.0 | 16.8 | 59 |



Figure 20. Targeting Error Histogram and Cumulative Distribution Function (cdf)
Figure 20 shows that while errors over 500 km occurred about $1.6 \%$ of the time, most errors were less than 50 km and the cases with the larger errors are few and far between. Many of these higher error cases resulted not from a lack of controllability but from deviations between the analytical and numerical solutions. The highest error run ( 1171 km total error) was one such case. In these Monte Carlo runs, targeting was set to begin when the spacecraft had 5400 seconds of longitude controllability. This would mean that through modulation of the ballistic coefficients, the spacecraft could extend the orbit lifetime by up to 5400 seconds without changing the impact latitude. The case with the 1171 km error was re-run with targeting starting at only 5000 seconds of longitude controllability. This means that the targeting begins closer to de-orbit, and because there is less remaining lifetime, the deviations between the analytical and numerical solutions are less. As a result, that simulation converged within an error of only 516 km despite the decrease in available longitude controllability. Because the guidance would be re-computed every few hours in practice, some other guidance errors may also be reduced as the satellite approaches de-orbit. If the targeting algorithm is run closer to the end of the orbit life, though there will be less discrepancy between the analytical and numerical solutions, controllability will suffer, especially longitude controllability. Table 2 shows that longitude error, on average, tends to be greater than latitude error due to the more limited longitude controllability. For cases where longitude control is not as important, targeting can begin significantly later in the orbit. This will significantly reduce simulation run time since each numerical propagation will require less time and the smaller deviations between the analytical and numerical solutions will mean that fewer iterations are required before convergence. As long as there is at least 42,000 seconds of lifetime controllability, the maximum theoretical longitude error will be about 1250 km for equatorial target points and will be less for higher latitude points.

Ultimately, the performance provided by this algorithm would be acceptable for the purposes of keeping spacecraft debris away from populated areas. Users can expect a targeting error of around 62 km , but can be reasonably sure that the spacecraft will not de-orbit more than 1500 km from the target point. According to section 4.7.2.1 of the NASA debris mitigation guidelines ${ }^{32}$, a selected trajectory for guided re-entry must ensure that no surviving debris impact with a kinetic energy greater than 15 joules is closer than 370 km from foreign landmasses, or is within 50 km from the continental U.S., territories of the U.S., and the permanent ice pack of Antarctica. Furthermore, the product of the probability of failure to track the guidance and the risk of human casualty associated with the failure must be less than $.0001(1: 10,000)$. In reality, a spacecraft would likely break apart and become a debris cloud upon reaching the reentry point. Mission designers must investigate the expected profile of the debris cloud and pick a target de-orbit point such that the sum of the maximum targeting error and the maximum spread of the debris cloud is sufficiently far from land. Since the most logical point to target for this purpose would be somewhere in the middle of the ocean, a de-orbit point can be easily selected to meet this requirement even with a guidance error of up to 1500 km .

## C. STK Validation of Algorithm and Propagator

To provide external validation for the MATLAB orbit propagator, a trajectory was propagated using the MATLAB propagator and the Astrogator module in AGI's Systems Tool Kit (STK) ${ }^{16}$. Both propagators used the NRLMSISE00 atmospheric model with time varying F10.7 and Ap and included zonal harmonic gravitational perturbations though J4. The oblateness of the Earth for altitude calculations was taken into account as was the rotation of the atmosphere

American Institute of Aeronautics and Astronautics
with Earth. An orbit simulation was run in both propagators with initial conditions $(a[k m], e, \Omega[\mathrm{deg}], \omega[\mathrm{deg}], \theta[\mathrm{deg}], i[\mathrm{deg}])=(6698,0,180,0,225,45)$, epoch $=\left[\begin{array}{ll}2004 & 51200] \text {, and control }\end{array}\right.$ parameters $\left(C_{b 1}, C_{b 2}, C_{b_{-} \text {term }}\right)=(.025, .025, .04) \mathrm{m}^{2} / \mathrm{kg}, t_{\text {swap }}=150,000 \mathrm{~s}$. Figure 21 shows the semi major axis over time for the STK and MATLAB propagators. The total percentage difference between the orbit lifetime predicted by each propagator was $2.79 \%$. This means that the difference in the average drag force experienced by the satellite in each propagator differed by $2.79 \%$; an amount easily correctable by an inner loop guidance tracker algorithm. These differences are likely because STK calculates altitude above the geoid for use in the density model while the MATLAB propagator calculates altitude above the WGS84 reference ellipsoid. While geodetic altitude is technically more correct, calculating altitude above the ellipsoid is much more computationally efficient and the resulting error is quite small. When the STK propagator was run with gravitational harmonics through degree 21 and order 21 and solar and lunar gravity included, the difference between the STK and MATLAB propagators was $2.92 \%$. This verifies that while the third body perturbations and the sectorial, tesseral and higher order zonal harmonics do make a difference, they are not significant perturbations and can be safely ignored for most applications.


Figure 21. Semi Major Axis over Time for MATLAB and STK Propagators

Several cases of the targeting algorithm were also run using STK's High Precision Orbit Propagator (HPOP). The HPOP utilizes the high fidelity EGM2008 gravitational model, takes into account solar and lunar gravity, and can use high fidelity atmospheric models. The HPOP was used with the NRLMSISE-00 density model with constant F10.7 and Ap indices to demonstrate the ability of the targeter to converge using a high fidelity simulation environment. The minimum and maximum ballistic coefficients for this simulation were .01 and .025 $\mathrm{m}^{2} / \mathrm{kg}$. The guidance generation sequence was the same as with the MATLAB propagator except that the new exponential atmospheric model was not created due to the difficulty of interfacing custom drag models with STK. All tested cases converged with the STK propagator with significantly lower run times than with the MATLAB propagator. This demonstrated that the algorithm was correct and robust enough to function in the highest fidelity simulation environments despite the simplifying assumptions made to develop the analytical solutions. The low run times with the STK propagator demonstrate that using an efficiently implemented propagator written in a compiled language such as c++ yields algorithm run times fast enough for onboard guidance generation every few hours to be feasible even with the limited computing power of the chips available for small spacecraft. One such chip is the Intel® Edison Compute Module ${ }^{33}$ which was found to be approximately 25 times slower at executing a simple for loop written in python than a 3.6 Ghz desktop workstation with a fourth generation intel i7. With the HPOP propagator, over half the algorithm run time was MATLAB computational overhead, so these times could be reduced further with a full c++ implementation. Table 3 shows the results of 5 targeting simulations run using the STK HPOP propagator.
Table 3. Results of Targeting Algorithm using STK HPOP Propagator

| Initial Orbital <br> Elements $(a[\mathrm{~km}], e$, <br> $\Omega[\mathrm{deg}], \omega[\mathrm{deg}]$, <br> $\theta[\mathrm{deg}], \mathrm{i}[\mathrm{deg}])$ | Epoch [y mdhms] | Target (lat[deg], long[deg]) | $\begin{gathered} \left(C_{b 1}, C_{b 2},\right. \\ \left.C_{b_{-}-t e r m}, t_{\text {swap }}\right) \end{gathered}$ | Number <br> Numerically <br> Propagated <br> Trajectories | $\begin{aligned} & \hline \text { Orbit } \\ & \text { Life } \\ & \text { (hours) } \end{aligned}$ | Total targeting error (km) | Simulation Run Time (s) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \left(6708,0,0,0,90^{\circ},\right. \\ \left.45^{\circ}\right) \end{gathered}$ | $\begin{gathered} {\left[\begin{array}{c} 201531 \\ 000 \end{array}\right]} \end{gathered}$ | $(20,60)$ | $\begin{gathered} (.0225, .0106, \\ .0175,143.5) \end{gathered}$ | 25 | 393.3 | 19.6 | 699 |
| $\begin{gathered} (6688, .004,0,0, \\ \left.45^{\circ}, 60^{\circ}\right) \end{gathered}$ | $\begin{gathered} {[201541} \\ 000] \end{gathered}$ | $(-30,40)$ | $\begin{aligned} & (.0128, .0120, \\ & .0175,140.1) \end{aligned}$ | 36 | 456.7 | 30 | 1174 |
| $\begin{gathered} \hline(6678,0,180,0, \\ \left.60^{\circ}, 90^{\circ}\right) \\ \hline \end{gathered}$ | $\begin{gathered} {[201551} \\ 000] \\ \hline \end{gathered}$ | $(10,200)$ | $\begin{gathered} (.0250, .0100, \\ .0175,73.6) \\ \hline \end{gathered}$ | 14 | 426.9 | 94 | 426 |
| $\begin{gathered} \left(6698,0,0,0,30^{\circ},\right. \\ \left.90^{\circ}\right) \end{gathered}$ | $\begin{gathered} {[201561} \\ 000] \end{gathered}$ | $(85,100)$ | $\begin{gathered} (.025, .01, \\ .0175,96.4) \end{gathered}$ | 10 | 518.1 | 83.2 | 391 |
| $\begin{gathered} \left(6698,0,90^{\circ}, 0,\right. \\ \left.45^{\circ}, 90^{\circ}\right) \\ \hline \end{gathered}$ | $\begin{gathered} {[201571} \\ 000] \\ \hline \end{gathered}$ | $(-60,0)$ | $\begin{gathered} (.0127, .0148, \\ .0175,215.8) \\ \hline \end{gathered}$ | 51 | 506 | 323.7 | 1796 |

American Institute of Aeronautics and Astronautics

## VII. Conclusions

Through simulations and mathematical analysis, the feasibility of targeting a de-orbit location with a spacecraft using solely aerodynamic drag has been demonstrated. An analytical solution was developed to estimate the ballistic coefficient profile necessary for a low Earth orbit spacecraft to de-orbit in the desired location. In the simplest case of a spherical Earth model with a standard non-rotating atmosphere, the analytical and numerical solutions matched almost exactly and the algorithm converged within a few iterations to an error under 10 km . In reality, however, using such a simplistic orbit model would result in a guidance that was a poor reflection of reality and would be difficult if not impossible to track. A high fidelity simulation environment was created taking into account a non-spherical Earth, atmospheric rotation, and NRLMSISE-00 density. The targeting algorithm was tested in this environment with 1000 Monte Carlo runs conducted using a set of randomized initial conditions. The algorithm converged for all cases and achieved targeting with under 1000 km error for all but one case which had an error of 1171 km .

Though running this algorithm using the high fidelity simulation environment in MATLAB took nearly an hour on average, performance improvements could be gained by re-writing this in c++, potentially decreasing run time by a factor of up to 500 according to some sources ${ }^{34}$ and facilitating guidance computation onboard a spacecraft. The Intel ${ }^{\circledR}$ Edison Compute Module ${ }^{33}$, which is small enough to fit inside a CubeSat ( $25 \times 35.53 .9 \mathrm{~mm}$ ), was found to be approximately 25 times slower than a high performance desktop PC. Despite this lower speed, the performance benefits from a compiled language such as c++ will mean that onboard chips such as the Edison will likely be able to generate guidances even more quickly than desktop workstations running MATLAB, facilitating periodic guidance re-computation and improved targeting accuracy. Additionally, for most spacecraft, the goal will be to target a point in the middle of the ocean to prevent damage to persons or property from falling debris. In this case, only the latitude targeting stage can be performed and the worst case longitude error will be about 1250 km though it will usually be much less. Such a longitude variation would still fulfill the NASA debris mitigation requirements ${ }^{32}$ and would greatly simply the algorithm, reduce computation time, and allow maneuvering to begin closer to the end of the orbit lifetime.

Overall, the algorithm performed satisfactorily in calculating the control parameters required to target a de-orbit point. In practice, a guidance would be created from a trajectory that was numerically propagated using these control parameters. The spacecraft ballistic coefficient would then be continuously modulated using attitude changes or the deploying/retracting of a drag device to ensure that the spacecraft followed that guidance. As such, the guidance does not need to be perfect but needs to be accurate enough that the spacecraft can track it. Generating the guidance using the high fidelity simulation environment provides this level of accuracy.

## VIII. Future Work

A natural continuation of this work is the development of the inner loop guidance tracking algorithm needed to ensure that the spacecraft actually follows the desired decay trajectory. The targeting algorithm discussed in this paper provides the $C_{b 1}, C_{b 2}$, and $t_{s w a p}$ values required to impact the earth in the desired location, but these values are based on a model containing numerous uncertainties, especially in the atmospheric density and drag coefficient. These uncertainties will mean that the satellite will not end up in the desired location if the ballistic coefficient control is applied open loop. For this reason, a feedback control loop will be necessary to ensure that the spacecraft tracks the guidance specified by the targeting algorithm which will be defined by values of orbital elements at each point in time. Because the uncertain orbital perturbations (such as aerodynamic drag) primarily affect in-plane motion, knowledge of the desired and actual true anomaly and rate of change of true anomaly should be sufficient to characterize the inplane error in the system. Based on the discrepancy between the desired and actual true anomaly and rate of change of true anomaly, a control algorithm could be designed to calculate how much the ballistic coefficient should be varied to help return the spacecraft to the guidance trajectory. Adaptive control methods may be utilized for this because the controller tuning will depend heavily on ambient density which can varying widely.

## Acknowledgements

The authors wish to thank AI Solutions for sponsoring this investigation under a NASA Kennedy Space Center's subcontract (project LSP 15-025: A Drag Device for Controlled Deorbiting of LEO Spacecraft). The authors also wish to thank Justin Treptow and Larry Fineberg from NASA, Scott Clark and Yusef Johnson from AI Solutions, and Dr. David C. Guglielmo from the University of Florida ADAMUS lab for their revision work and valuable suggestions.

## IX. References

1 Patera, R. P., and Ailor, W. H., "The realities of reentry disposal," Proceedings of the AAS/AIAA Space Flight Mechanics Meeting, 1998, pp. 9-11.
2 Ailor, W. H., and Patera, R. P., "Spacecraft re-entry strategies: Meeting debris mitigation and ground safety requirements," Proceedings of the Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering, vol. 221, Jun. 2007, pp. 947-953, DOI: 10.1243/09544100JAERO199.
3 Heiney:KSC, A., "NASA - Landing 101" Available: http://www.nasa.gov/mission_pages/shuttle/launch/landing101.html.
4 National Aeronautics and Space Administration, "Guidance and Navigation for Entry Vehicles," 1968.
5 Moore, T. E., Space shuttle entry terminal area energy management, 1991.
6 Heidt, H., Puig-Suari, J., Moore, A., Nakasuka, S., and Twiggs, R., "CubeSat: A New Generation of Picosatellite for Education and Industry Low-Cost Space Experimentation," AIAA/USU Conference on Small Satellites, Aug. 2000.
7 Skrobot, G. L., and Coelho, R., "ELaNa-Educational Launch of Nanosatellite Providing Routine RideShare Opportunities," 2012.

8 Moe, K., and Moe, M. M., "Gas-surface interactions and satellite drag coefficients," Planetary and Space Science, vol. 53, Jul. 2005, pp. 793-801, DOI: 10.1016/j.pss.2005.03.005.
9 Pérez, D., and Bevilacqua, R., "Neural Network based calibration of atmospheric density models," Acta Astronautica, vol. 110, May 2015, pp. 58-76, DOI: 10.1016/j. actaastro.2014.12.018.
10 Omar, S. R., and Wersinger, J. M., "Satellite Formation Control using Differential Drag," 53rd AIAA Aerospace Sciences Meeting, American Institute of Aeronautics and Astronautics, .
11 Pérez, D., and Bevilacqua, R., "Differential Drag-Based Reference Trajectories for Spacecraft Relative Maneuvering Using Density Forecast," Journal of Spacecraft and Rockets, vol. 53, Jan. 2016, pp. 234-239, DOI: 10.2514/1.A33332.
12 Pastorelli, M., Bevilacqua, R., and Pastorelli, S., "Differential-drag-based roto-translational control for propellant-less spacecraft," Acta Astronautica, vol. 114, Sep. 2015, pp. 6-21, DOI: 10.1016/j. actaastro.2015.04.014.
13 Josep Virgili, P. C. E. R., "Atmospheric Interface Reentry Point Targeting Using Aerodynamic Drag Control," Journal of Guidance, Control, and Dynamics, vol. 38, 2015, pp. 1-11, DOI: 10.2514/1.G000884.
${ }^{14}$ Guglielmo, D., and Bevilacqua, R., "Propellant-less Atmospheric Differential Drag LEo Spacecraft (PADDLES) Mission," SmallSat Conference, 2014.
15 Mason, C., Tilton, G., Vazirani, N., Spinazola, J., Guglielmo, D., Robinson, S., Bevilacqua, R., and Samuel, J., "Origami-based Drag Sail for CubeSat Propellant-free Maneuvering," Tokyo, Japan: 2013.
16 Analytical Graphics Inc, Systems Toolkit.
17 Vallado, D., Fundamentals of Astrodynamics and Applications, Hawthorne, CA: Microcosm Press, 2013.
18 Curtis, H., Orbital Mechanics for Engineering Students, Elsevier, 2009.
19 International Organization for Standardization, "ISO/DIS 14222, Space Environment (natural and artificial) - Earth Upper Atmosphere," Sep. 2013.
${ }^{20}$ Franklin, G., Powell, J., and Emami-Naeini, A., Feedback Control of Dynamic Systems, Prentice Hall, 2002.
21 Madden, M. M., "Gravity Modeling for Variable Fidelity Environments," AIAA Modeling and Simulation Technologies Conference and Exhibit, 2006, p. 6730.
22 Pavlis, N. K., Holmes, S. A., Kenyon, S. C., and Factor, J. K., "An Earth Gravitation Model to Degree 2160: EGM2008," Apr. 2008.
${ }^{23}$ Bate, R. R., Mueller, D. D., and White, J. E., Fundamentals of astrodynamics, New York: Dover Publications, 1971.
${ }_{24}$ Karney, C., GeographicLib, 2016.
25 Vallado, D. A., and Finkleman, D., "A critical assessment of satellite drag and atmospheric density modeling," Acta Astronautica, vol. 95, Feb. 2014, pp. 141-165, DOI: 10.1016/j. actaastro.2013.10.005.
26 MathWorks, "MathWorks atmosnrlmsise00 Documentation" Available: http://www.mathworks.com/help/aerotbx/ug/atmosnrlmsise00.html.
27 NASA Goddard Space Flight Center, "OMNIWeb" Available: http://omniweb.gsfc.nasa.gov/form/dx1.html.
28 National Oceanic and Atmospheric Administration, "USAF 45-Day Ap and F10.7cm Flux Forecast" Available: http://www.swpc.noaa.gov/products/usaf-45-day-ap-and-f107cm-flux-forecast.
${ }^{29}$ Marcos, F., Bowman, B., and Sheehan, R., "Accuracy of Earth's thermospheric neutral density models," 2006.
${ }^{30}$ King-Hele, D., "Decrease In Upper Atmosphere Rotation Rates," Letters to Nature, 1971.
${ }^{31}$ Kohl, H., and King, J. W., "Atmospheric winds between 100 and 700 km and their effects on the ionosphere," Journal of Atmospheric and Terrestrial Physics, vol. 29, Sep. 1967, pp. 1045-1062, DOI: 10.1016/0021-9169(67)90139-0.
32 National Aeronautics and Space Administration, "Process for Limiting Orbital Debris," May 2012.
33 "Intel® Edison-One Tiny Module, Endless Possibility," Intel Available: http://www.intel.com/content/www/us/en/do-ityourself/edison.html.
34 Andrews, T., "Computation time comparison between matlab and C++ using launch windows," 2012.


[^0]:    ${ }^{1} \mathrm{PhD}$ Student, Department of Mechanical and Aerospace Engineering, MAE-A Room 211, AIAA Student Member.
    ${ }^{2}$ Associate Professor, Department of Mechanical and Aerospace Engineering, MAE-A Room 308, AIAA Senior Member.

