Nonlinear Attitude Control of Satellite Platforms equipped with Variable Speed Control Moment Gyroscopes

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Variable Speed Control Momentum Gyroscopes are single-gimballed gyroscopes where the fly wheel speed can vary. The equations of motion of a generic rigid satellite equipped with V.S.C.M.G. are developed and presented in a ready-to-be-implemented form. Based on these equation a MATLAB Toolbox named "V.S.C.M.G. Toolbox" has been programmed and is here used to model the non linear control system of a spacecraft equipped with a generic "fly-wheel" system. Reaction Wheels, Control Moment Gyroscopes and Variable Speed Control Moment Gyroscopes might be simulated in any number by means of an elementary block connection. A non linear feedback, based on the so called "velocity based" steering law, is also contained in the toolbox so that the complete control system might be simulated. To alleviate the shortcomings of this particular a non linear controller a parameter α has been introduced. Such a parameter reveals to have a twofold importance: in a "velocity based" type of controller it tells us how good the approximations made were, and in a generic manoeuvre it gives us an idea on how good the steering logic used is. In other words it tells us whether we are exploiting the torque amplification effects or not. A numerical example is given at the end.

Section 1: Introduction

There is more than one way to actuate the torque demand of a spacecraft attitude control system. The most popular methods are the employment of jet thrusters and the use of rotating wheels (fly-wheels). The first system has the advantage of being usually very straight forward to design and quite effective. Nevertheless the impact of the tank plus pipes system on the overall design is quite large and the life of these devices is anyway limited by fuel consumption. Besides the plumes of the thrusters could impinge on some external payloads, such as communications devices or optical instruments, compromising the design of the entire mission. On the other hand fly-wheels actuators have the great advantage of being completely contained inside a small volume inside the spacecraft structure and of using energy provided directly by the power subsystem rather than consuming an exhaustible resource. The drawback is that the non-linear dynamic of these systems makes it quite difficult to find a feedback control law and, even when this is possible, the control design can still be quite complicated. Fly-wheels systems capable of actuating the control torque demand are commonly divided into three main categories: Reaction Wheels (RW), Control Moment Gyros (CMG) and Variable Speed Control Moment Gyros (VSCMG). The first system is a set of wheels rotating with variable speed around their spin axis fixed with respect to the spacecraft reference frame. The variation of their rotational speed creates a torque around the spin axis, whereas the speed itself introduces some gyro effects that have to be accounted for when the spacecraft is undergoing a non null motion. Control Moment Gyros are similar devices in which the wheels rotate at a constant velocity, but are mounted on gimbals that are capable of rotating around their axis (fixed in the spacecraft reference frame). The rotation speed around the gimbals introduces a torque perpendicular to the wheel spin axis and to the gimbals axis. In some literature gimbals capable to rotate around two different axis are also considered so that the acronym SGCMG is sometimes used to denote Single Gimbal Control Moment Gyros. CMG suffer from an intrinsic problem, called the singularity problem, linked to the difficulty of exerting torques in certain directions for some configuration that can always be reached regardless of the number and geometry of the CMG mounted. Variable Speed Control Moment Gyros combine the two previous systems (see their scheme in figure 1) allowing the spin velocity of the wheels in a CMG system to vary with time. RW have been used in the past to perform accurate attitude tracking for satellites, the torque created by a wheel speed change is quite small and wheels speed saturation problems can arise. Due to the so called torque amplification effect CMG systems are more effective in this respect, but create the singularity regions that, even when an avoidance strategy is adopted, can lead to inaccurate tracking. Large structures are anyway considered to carry these devices (SKYLAB, ISS) due to their quoted torque amplification properties. Recent works by Tsiotras et al. [5] suggest that the extra degrees of freedom introduced in a VSCMG system could be used to realize an Integrated Power Attitude Control System (IPACS), a concept that, even if of quite an old conception, has never been implemented due to the difference between the required fly-wheel speed (40K-80K Round Per Minute RPM) and the common CMG speed (5K RPM). Thanks to the great advances in materials strength, mainly accomplished through composite structures, it is now feasible to think about these systems. A control law design

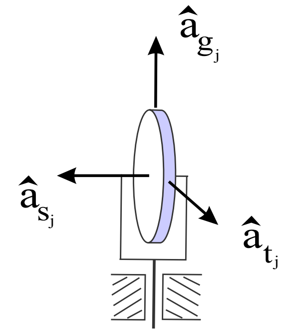

Fig.1: A V.S.C.M.G. and its relative reference frame.

process is introduced by Vadali et al. [1] by means of a Lyapunov function, an accomplishment that limits the shortcomings of a highly non linear dynamic. The steering law there introduced, named "velocity based" is based upon an assumption that lessen its generality. An alternative approach would be that of considering a "torque based" law such as that tried by Izzo and Valente [3] trying to actually drive the VSCMG mainly as CMG, or to try a different approach such as that introduced by Avanzini and de Matteis [4].

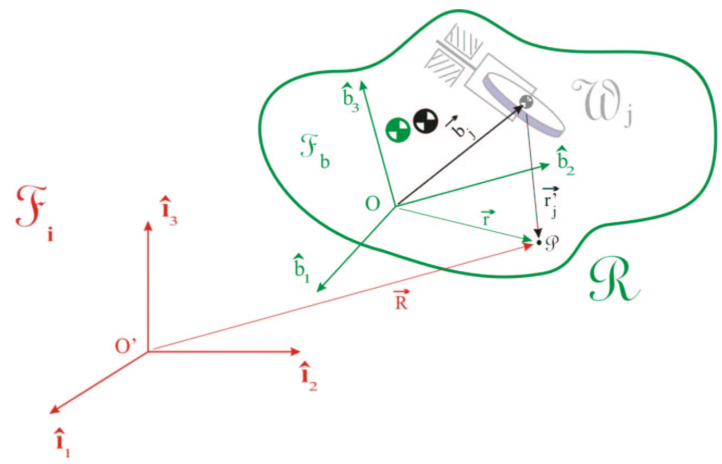

Fig.2: Definition of some relevant quantities of the mathematical model.

Section 2: The mathematical model

Dynamics.

The full mathematical model describing the dynamic of a system of n VSCMG and one satellite platform, all considered to be rigid bodies, is here developed. Different models (differing mainly in the notation) are already present in literature but we felt that a ready-to-be-implemented kind of derivation was necessary at this stage. Furthermore the dynamic of the wheels and of the gimbals is often left out the model and anyway not developed systematically. Let us start by writing the balance of absolute angular momentum for the whole body $\mathcal{B} = \mathcal{R} \bigcup_j \mathcal{W}_j$. We have $\dot{\vec{\mathbf{h}}}_O + \vec{\mathbf{v}}_O \times \vec{\mathbf{p}} = \vec{\mathbf{g}}_O$

where:

$\vec{\mathbf{p}} := \int_{\mathcal{B}} \rho \vec{\mathbf{v}} dV$	Total momentum of the system.
$\vec{\mathbf{h}}_O := \int_{\mathcal{B}} \rho \vec{\mathbf{r}} \times \vec{\mathbf{v}} dV$	Absolute angular momentum of the system with respect to the point O.
$\vec{\mathbf{v}}_O$	Inertial velocity of the point O.
$\vec{\mathbf{g}}_O$	Sum of all the external force moments acting on the system. The moments are evaluated with respect to O.

By manipulating these expression, introducing the second moment of inertia dyadics of the platform with respect to O $\vec{\mathbf{J}}_b := \int_{\mathcal{R}} \rho \left(r^2 \vec{\mathbf{1}} - \vec{\mathbf{r}}\vec{\mathbf{r}} \right) dV$ and of the j-th wheel plus gimbal structure with respect to its center of mass $\vec{\mathbf{Y}}_j := \int_{\mathcal{W}_j} \rho \left(r'^2 \vec{\mathbf{1}} - \vec{\mathbf{r}}'\vec{\mathbf{r}}' \right) dV$ it is possible to show that the absolute angular momentum with respect to the center of mass of the whole system can be put in the form:

$$\vec{\mathbf{h}}_{\oplus_T} = \left(\vec{\mathbf{J}} + \sum_{j=1}^{n} \vec{\mathbf{Y}}_j \right) \cdot \vec{\omega} + \sum_{j=1}^{n} Y_{g_j} \dot{\gamma}_j \hat{\mathbf{a}}_{g_j} + \sum_{j=1}^{n} I^w_{s_j} \Omega_j \hat{\mathbf{a}}_{s_j} \tag{1}$$

where $\vec{\mathbf{J}} = \vec{\mathbf{J}}_b + \sum_{j=1}^{n} m_j \left(b_j^2 \vec{\mathbf{1}} - \vec{\mathbf{b}}_j \vec{\mathbf{b}}_j \right)$ is the sum of the second moment of inertia dyadic of the platform and a term representing the second moment of inertia of a system of point masses (with mass equal to the mass of the j-th VSCMG) concentrated in the centre of masses of the VSCMG devices. Also $Y_{g_j} = \vec{\mathbf{Y}}_j \cdot \hat{\mathbf{a}}_{g_j}$ and $I_{s_j}^w$ is the sole wheel spin inertia. We have here also introduced the important quantities γ_j and Ω_j that represent the gimbal angles and the wheels speed. The angular velocity $\vec{\omega}$ is that of the platform $\Re$. If we perform the derivative and we project the whole equation on the body frame, the following final equation is obtained:

$$
\begin{aligned}
&\left(\mathbf{J} + \sum \mathbf{C}_j \mathbf{Y}_j \mathbf{C}_j^T\right)\dot{\boldsymbol{\omega}} + \sum Y_{g_j} \ddot{\gamma}_j \mathbf{g}_j + \sum I_{s_j}^w \dot{\Omega}_j \mathbf{s}_j + \sum I_{s_j}^w \Omega_j \dot{\gamma}_j \mathbf{t}_j + \\
&+ \sum \left(Y_{s_j} - Y_{t_j}\right)\dot{\gamma}_j \left(\mathbf{t}_j \mathbf{s}_j^T + \mathbf{s}_j \mathbf{t}_j^T\right)\boldsymbol{\omega} + \boldsymbol{\omega}^x \left(J + \sum \mathbf{C}_j \mathbf{Y}_j \mathbf{C}_j^T\right)\boldsymbol{\omega} + \\
&+ \sum Y_{g_j} \ddot{\gamma}_j \boldsymbol{\omega}^x \mathbf{g}_j + \sum I_{s_j}^w \dot{\Omega}_j \boldsymbol{\omega}^x \mathbf{s}_j = \mathbf{g}_b
\end{aligned}
\tag{2}
$$

where all the bold quantities represent vector components in the body frame or matrices. In particular $\mathbf{C}_j = (\mathbf{g}_j \mid \mathbf{s}_j \mid \mathbf{t}_j)$ is the matrix that allow to pass from the body frame to the frame relative to the *j-th* VSCMG. Equations (2) may be used as the basis in all the simulations involving RW, CMG or VSCMG. To complete the model the dynamics of the gimbals and of the wheels have to be investigated. If we write the balance of the absolute angular momentum for the gimbals and for the wheels and projecting into the body frame, the following might be obtained:

$$
\begin{aligned}
&Y_{g_j}\left(\dot{\boldsymbol{\omega}} \cdot \mathbf{g}_j + \ddot{\gamma}_j\right) + \left[\left(Y_{t_j} - Y_{s_j}\right)\left(\boldsymbol{\omega} \cdot \mathbf{s}_j\right) - I_{s_j}^w \Omega_j\right]\left(\boldsymbol{\omega} \cdot \mathbf{t}_j\right) = G_j \\
&I_{s_j}^w\left(\dot{\boldsymbol{\omega}} \cdot \mathbf{s}_j - \gamma_j\left(\boldsymbol{\omega} \cdot \mathbf{t}_j\right) + \Omega_j\right) = S_j
\end{aligned}
\tag{3}
$$

where G_j and S_j are the torque exerted on the gimbals axis and on the spin axis.

Kinematics.

We will not discuss here on the advantages that a given attitude representation brings to the analytical properties of the kinematics relations. We address to the work of Shaub and Junkins [7] and Shuster [8] for a complete review on this subject. In this work the Modified Rodriguez Parameters (MRPs) have been chosen to describe the attitude of the rigid platform $\Re$. We shortly recall that these parameters are defined in terms of the rotation eigenaxis $\hat{\mathbf{e}}$ and the rotation angle ϕ (the rotation we are referring to is granted to exist thanks to Euler's theorem):

$$
[\sigma_1, \sigma_2, \sigma_3]^T := \boldsymbol{\sigma} := \hat{\mathbf{e}} \tan\left(\frac{\phi}{4}\right) \tag{4}
$$

It is easy to show that the kinematics equation in terms of MRPs takes the form:

$$
\dot{\boldsymbol{\sigma}} = \tfrac{1}{2}\left(\boldsymbol{\sigma}^x + \boldsymbol{\sigma}\boldsymbol{\sigma}^T + \frac{1 - \boldsymbol{\sigma}^T \boldsymbol{\sigma}}{2}\mathbf{I}\right)\boldsymbol{\omega} = \mathbf{G}(\boldsymbol{\sigma})\boldsymbol{\omega} \tag{5}
$$

The problems arising when singular configurations are reached ($\phi = \pm\pi$) are easily avoided through the use of a so called "shadow set", in the calculations involved in this work these configurations are never reached and the shadow set could not be implemented.

Section 3: The control problem

The mathematical model being established the focus will now be on the control issue. In particular lets suppose that a given attitude history wants to be tracked. The desired MRPs and angular velocity of the platform will be denoted by the symbols $\boldsymbol{\sigma}_d$, $\boldsymbol{\omega}_d$. The angular velocity error is then defined as the relative angular velocity of the platform as seen by the desired orientation, i.e. $\boldsymbol{\omega}_e = \boldsymbol{\omega} - \boldsymbol{\omega}_d$. The attitude error is defined as the MRPs that describe the rotation needed to overlap the desired attitude to the actual attitude. We therefore have the differential definition $\dot{\boldsymbol{\sigma}}_e = \mathbf{G}(\boldsymbol{\sigma}_e)\boldsymbol{\omega}_d$. In order to design a non linear feedback via a Lyapunov approach the following function (see Tsiotras [5]) is introduced:

$$V(\boldsymbol{\sigma}_e, \boldsymbol{\omega}_e) = \frac{1}{2}\boldsymbol{\omega}_e^T\left(\mathbf{J} + \sum \mathbf{C}_j\mathbf{Y}_j\mathbf{C}_j^T\right)\boldsymbol{\omega}_e + 2k_0 \ln\left(1 + \boldsymbol{\sigma}_e^T\boldsymbol{\sigma}_e\right) \tag{6}$$

where $k_0 > 0$ is a positive constant. This function is radially unbounded and positive definite. To get a global asymptotically stable feedback it is therefore sufficient to impose to its derivate to be negative definite. Such an imposition leads to the following condition:

$$\begin{aligned}&\sum\left[\tfrac{1}{2}\left(Y_{s_j}-Y_{t_j}\right)\left(\mathbf{t}_j\mathbf{s}_j^T+\mathbf{s}_j\mathbf{t}_j^T\right)\left(\boldsymbol{\omega}+\boldsymbol{\omega}_d\right)+I_{s_j}^w\Omega_j\mathbf{t}_j+Y_{g_j}\boldsymbol{\omega}^x\mathbf{g}_j\right]\dot{\gamma}_j+\\&+\sum Y_{g_j}\ddot{\gamma}_j\mathbf{g}_j+\sum I_{s_j}^w\dot{\Omega}_j\mathbf{s}_j=\\&=K\boldsymbol{\omega}_e+k_0\boldsymbol{\sigma}_e-\left(\mathbf{J}+\sum\mathbf{C}_j\mathbf{Y}_j\mathbf{C}_j^T\right)\dot{\boldsymbol{\omega}}-\boldsymbol{\omega}^x\left(\mathbf{J}+\sum\mathbf{C}_j\mathbf{Y}_j\mathbf{C}_j^T+\sum I_{s_j}^w\Omega_j\mathbf{s}_j\right)+\mathbf{g}_b\end{aligned} \tag{7}$$

that may be written in the following two equivalent forms:

$$\begin{aligned}&\mathbf{B}\ddot{\boldsymbol{\gamma}} + \mathbf{C}\dot{\boldsymbol{\gamma}} + \mathbf{D}\dot{\boldsymbol{\Omega}} = \mathbf{L}_r\\&\sum \mathbf{g}_j G_j + \sum \mathbf{s}_j S_j = \mathbf{T}_r\end{aligned} \tag{8}$$

the first one of which was developed by Tsiotras, the second by Izzo. We have introduced a number of new matrices and vectors that will not be here defined as not to obscure the discussion. If the torques G_j and S_j (*or equivalently the gimbals angles accelerations their velocities and the wheels accelerations*) applied by the electrical motors to the gimbals and to the wheels do satisfy the second (*or the first one*) of these relations, then the error would tend to zero and the attitude would be tracked. There are infinite ways of satisfying these relations, therefore infinite strategies we have to choose in between. As it has been shown by Izzo and Valente [3] such a choice is crucial as it may lead to unpractical control laws. In particular the greatest care has to be put in the exploitation of the torque amplification effect, that is in the use of the gimbals angle rate of change to build up the required torque. Being this issue still quite opened we will make use here of the so called "velocity based" control law as essentially presented by Vadali et al. [6]. Such a strategy is based, on the initial arbitrary assumption that the term $\mathbf{B}\ddot{\gamma}$ in eq.(8) is negligible. To verify that this is true we introduce here the parameter:

$$\alpha := \frac{|\mathbf{B}\ddot{\gamma}|}{|\mathbf{L}_{rm}|} \tag{9}$$

representing the relative percentage of the neglected term over the whole required control torque. As soon as this parameter becomes larger than $1/20$ (0.05%) the simulation will be considered to be unreliable. This criteria, though arbitrary has revealed to be quite satisfactory for a great number of manoeuvres.

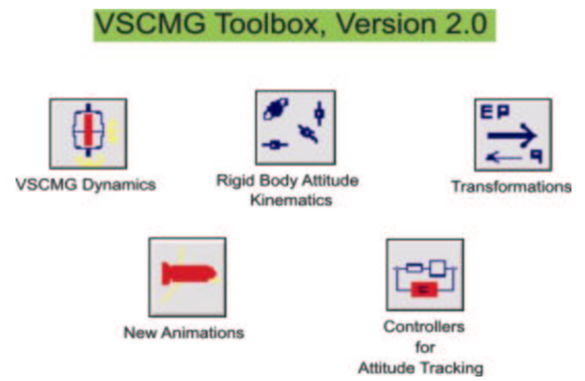

Fig.3: The main groups of VSCMG Toolbox V 2.0

Section 4: V.S.C.M.G. Toolbox V2.0

The ready-to-be-implemented equations presented in the previous sections have been used to program a SIMULINK library named VSCMG Toolbox V2.0 whose aim is to easily simulate the control system of any spacecraft equipped with a generic number of actuators (RW, CMG, VSCMG).

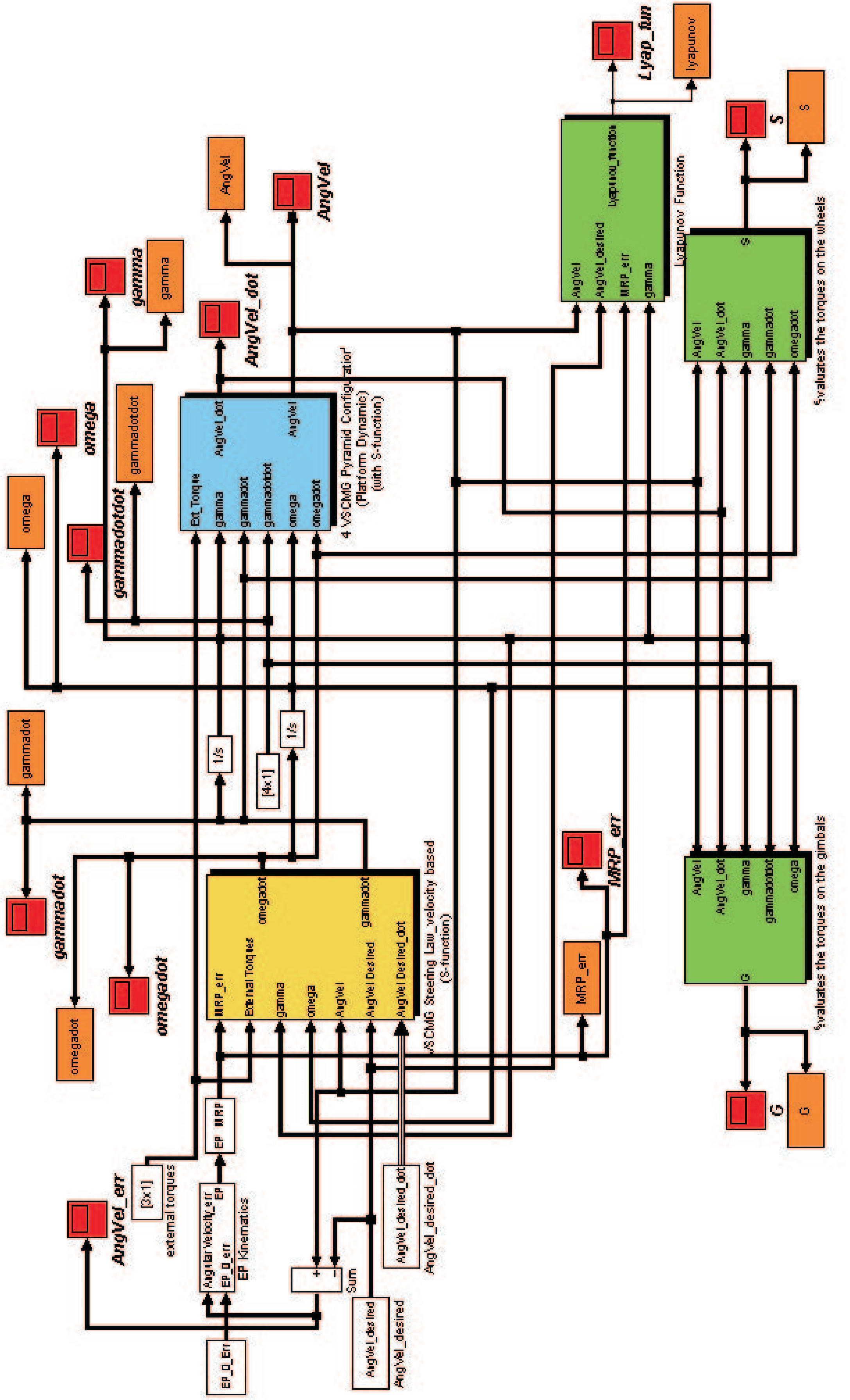

Fig.4: An example of a spacecraft control system simulator built connecting and grouping the blocks contained in the VSCMG Toolbox V2.0.

The Toolbox programming was started in June 2002 in collaboration with **Cranfield University** and **Analyticon Limited**. Since then it has been enriched by the contribution of students and researchers of the **University "La Sapienza"** in Rome. In its current version the toolbox is able to simulate the most common steering control laws for VSCMG, CMG and RW. Any configuration may be easily simulated thanks to the use of flexible blocks that, properly connected, build up the final spacecraft control system. A visualization of the GUI of the Toolbox is given in figure 3 as an example. The blocks programmed are divided into five major groups: **VSCMG dynamics**, **Rigid Body Attitude Kinematics**, **Transformations**, **New Animations**, **Controllers for Attitude Tracking**. As shown in figure 4 the various blocks may be connected as to build up a model simulating the spacecraft behaviour. Let us comment briefly on the example shown in figure 4. It basically consist of a model simulating the velocity based control system of a satellite platform equipped with a pyramidal system of VSCMG. The geometrical configuration of the device is shown in figure 5. The core blocks of the model are "4 VSCMG Pyramid Configuration (Platform Dynamic)" that basically solves eq.(2) for the chosen geometry and "VSCMG steering law velocity based" that implements the velocity based philosophy for controlling fly wheels devices. The other blocks serve just to get the desired output and to perform a bit of post processing. This is obviously just an example of how the VSCMG Toolbox V2.0 might be used, virtually any kind of spacecraft control system may be simulated and any new kind of control strategy may be easily added to the Toolbox. It is one of the future possible developments of this software to include blocks representing sensors and noise.

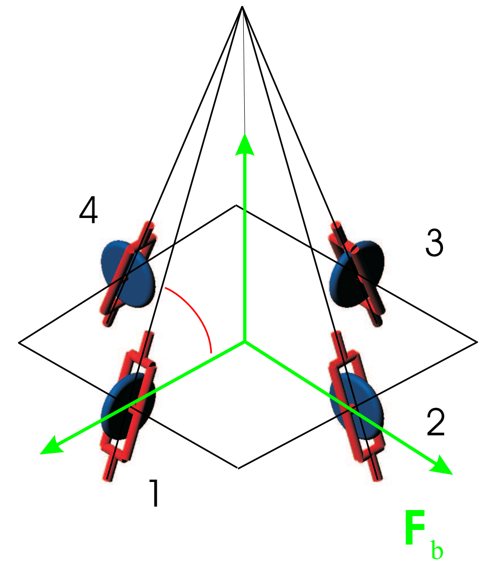

Fig.5: A VSCMG pyramidal configuration.

Section 5: Some simulations and results.

To show how the design of the control parameters in a velocity based strategy implies a trial and error kind of design for each particular manoeuvre (due to the approximation introduced when neglecting the term $\mathbf{B}\ddot{\gamma}$) we will here show some numerical simulations on a slew manouvre. The data used in the simulation are summarized in the tables below.

Symbol	Value	Units
θ	54.75	Deg.
$\mathbf{J}$	$\begin{bmatrix} 86.215 & 0 & 0 \\ 0 & 85.07 & 0 \\ 0 & 0 & 113.565 \end{bmatrix}$	$kg\ m^2$
$\mathbf{Y}_j$	$\begin{bmatrix} 0.13 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.03 \end{bmatrix}$	$kg\ m^2$
$I_{s_j}^w$	0.1	$kg\ m^2$
Y_{g_j}	0.03	$kg\ m^2$
$\mathbf{\Omega}(0)$	$[14 \quad 14 \quad 14 \quad 14]^T$	$\frac{rad.}{\text{sec.}}$
W_{s0}	2	
W_g	1	

Symbol	Value	Units
$\boldsymbol{\gamma}(0)$	$[0 \quad 0 \quad \frac{\pi}{2} \quad -\frac{\pi}{2}]^T$	$rad.$
$\dot{\boldsymbol{\gamma}}(0)$	$[0 \quad 0 \quad 0 \quad 0]^T$	$\frac{rad.}{\text{sec.}}$
$\boldsymbol{\omega}(0)$	$[0.01 \quad 0.05 \quad -0.01]^T$	$\frac{rad.}{\text{sec.}}$
$[\boldsymbol{\varepsilon}(0), \eta(0)]$	$[0.63 \quad 0.46 \quad 0.30 \quad 0.53]^T$	-
$\mathbf{g}_b$	$[0 \quad 0 \quad 0]^T$	Nm
μ	10^{-9}	$\frac{rad.}{\text{sec.}}$
k_0	1.7	
$\mathbf{K}$	$\begin{bmatrix} 13.13 & 0 & 0 \\ 0 & 13.04 & 0 \\ 0 & 0 & 15.08 \end{bmatrix}$	

The desired angular velocity and MRPs are those relative to a slew manoeuvre (in particular it is supposed that the angular velocity on the roll axis is a sine with period 30 sec. and the other components are zero). The various control parameters were chosen to meet specific requirements on the overshoot and control speed. The results of the simulation are, in terms of the Lyapunov function, the MRPs error and the angular velocity error, shown in figure 6. The trend of the parameter α is also shown. Its trend, for the 14rpm simulation, shows how the gimbals acceleration reached high values in correspondence with singular configurations, resulting in an unacceptable control law. The magnitude of the parameter α has a twofold importance. In a velocity based type of controller it tells us how good the approximations made were, and in a generic VSCMG manoeuvre it gives us an idea on how good our steering law was, that is, on how much we managed to exploit the torque amplification effects of the "fly-wheel" device. For these reasons we have to consider the simulation as erroneous whenever α exceeds the chosen value. In the specific case of this slew manoeuvre the satellite is unable to perform it using a velocity based steering law. The only hypothetical solution, would be, in this case, to speed up the wheels as to allow for a better use of the CMG advantages.

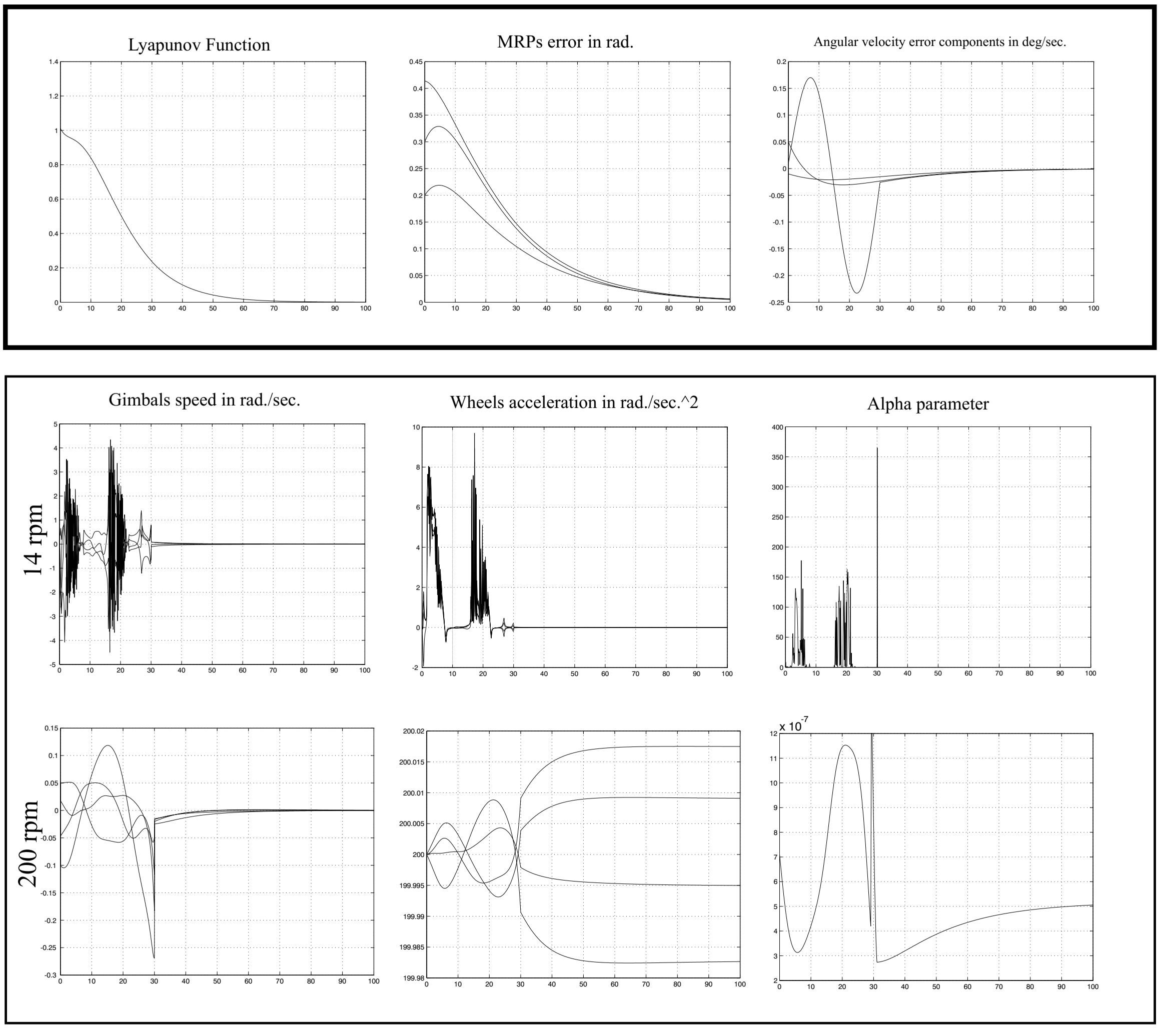

Fig.6: The outcome of the numerical simulation for the slew manoeuvre with initial wheel speed of 14 rpm and 200 rpm. Only the second manoeuvre might be considered feasible as the values assumed by the parameter α remain limited.

For this reason a second simulation was run with starting wheel velocities of 200 rpm (a speed-up manoeuvre would then be required before starting the slew). The outcome is satisfactory, except for a peak of the parameter α in the neighbourhood of 30 sec. that is anyway due to a discontinuity of the input signal and has therefore been considered as unavoidable.

Section 6: Conclusions.

A MATLAB Toolbox has been programmed with the aim of providing to the engineering community a useful tool to build up easily a precise simulation of the control system of a spacecraft equipped with actuating fly-wheels. The steering law "velocity based" (see [6]) is implemented in the toolbox. Whether to solve the issue on the validity of such a control strategy based on an a priori assumption, a parameter α is introduced measuring both the magnitude of the approximations made and the good quality of the control law, that is the good exploitation of the torque amplification effect.

References:

[1] H.S. Oh and S.R. Vadali, Feedback Control and Steering Laws for Spacecraft Using Single Gimbal Control Moment Gyros. *The Journal of the Astronautical Sciences*, Vol.39, No.2, April-June 1991, pp. 183-203.

[2] K.A. Ford and C.D. Hall, Singular Direction Avoidance Steering for Control Moment Gyros. *Journal of Guidance, Control and Dynamics,* Vol. 23, No. 4, July-August 2000.

[3] D. Izzo and C. Valente, A Globally Stable Lyapunov Non Linear Controller for the Attitude of a platform equipped with a system of actuating gyroscopes. *Wascom International Conference,* Villasimius (Cagliari, Italy), June 1-7, 2003

[4] G.Avanzini and G. de Matteis, A Local Optimization Technique for Attitude Motion Tracking Using Control Moment Gyros, *proceedings of the 11th AAS/AIAA Space Flight Mechanics Meeting,* 2001, pp.109-123.

[5] H. Yoon, P.Tsiotras, Spacecraft Adaptive Attitude And Power Tracking With Variable Speed Control Moment Gyroscopes. *AAS/AIAA Astrodynamics Conference* , Quebec City, Canada, July 30-August 2, 2001.

[6] H. Schaub, S.R. Vadali and J.L.Junkins, Feedback control law for variable speed control moment gyros. *Journal of the Astronautical Science*, Vol.44, No.1, 1996.

[7] H. Schaub and J.L.Junkins. Stereographic Orientation Parameters for Attitude Dynamics: a generalization of Rodriguez Parameters. *Journal of the Astronautical Science*, Vol.44, No.1, 1996.

[8] M.D. Shuster, Stereographic A Survey of Attitude Representations. *Journal of the Astronautical Science*, Vol.41, No.4, 1993.