# Rendezvous Maneuvers of Multiple Spacecraft using Differential Drag under $\mathbf{J}_{2}$ Perturbation 

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#### Abstract

In this work the residual atmospheric drag is exploited to perform rendezvous maneuvers among multiple spacecraft in low Earth orbits. These maneuvers are required, for instance, for autonomous on-orbit assembly. By varying the level of aerodynamic drag of each spacecraft, relative differential accelerations are generated among the spacecraft of the group, and, therefore, their relative orbits are controlled. Each of the spacecraft is assumed to include a drag plate, which can be actively opened or closed, in order to vary the atmospheric drag. The recently developed Schweighart-Sedwick model is used to describe the relative dynamics of different spacecraft with respect to a circular orbit with the inclusion of $\mathbf{J}_{2}$ effects. Furthermore, the natural relative dynamics of each chaser with respect to the target is decoupled into a secular motion and a periodic oscillation. In particular, the following two-phase control method is proposed. First the secular motion of each chaser is controlled via differential drag in order for the spacecraft to sequentially move from an arbitrary initial condition to a closed stable relative orbit around the target spacecraft. After the relative orbit stabilization, a relative eccentricity control is applied to each spacecraft in order to zero out the semi-axis of the relative orbit around the target and achieve the rendezvous condition. The control algorithm considers mutual constraints among the values of differential drag that the different spacecraft can experience. Potential collisions are avoided by changing the maneuvering initial time. The main advantage of the proposed technique is that it enables a fleet of spacecraft to rendezvous without propellant expenditure. Furthermore, no numerical optimization is needed, as the control policy is based on closed-form analytical solutions. The proposed technique was validated via numerical simulations.


## Nomenclature

$a \quad=\quad$ Magnitude of Differential Drag
$a_{\text {ISS }}=$ International Space Station Orbit's Semi-major Axis
$A=$ Constant Coefficient in State Matrix for the Transformed State Vector
$B \quad=\quad$ Constant Coefficient in State Matrix for the Transformed State Vector
$a_{\text {orb }}=$ Orbit Semi-Major Axis
c $=$ Coefficient in Schweighart-Sedwick Equations
$C_{D}=$ Drag Coefficient
$\Delta E=$ Energy Dead-Band
$\Delta x, \Delta y, \Delta z=$ Mutual Position Coordinates of Two Spacecraft in LVLH
$\Delta \dot{x}, \Delta \dot{y}, \Delta \dot{z}=$ Mutual Velocity Components of Two Spacecraft in LVLH
$\Delta t^{*}=$ Unknown Time Duration for the Controlled Rendezvous Phases
$\Delta t_{w}=$ Waiting Time Interval before Controlled Phase in Rendezvous Maneuver
$\Delta t_{s}=$ Sample Time Interval

[^0]$d=$ Nonlinear Function of $\Delta t^{*}$ for computing Rendezvous Maneuver Time Duration
$d_{c}=$ Tolerance Distance for Collision Avoidance
$e=$ Time-Varying Eccentricity of the Harmonic Oscillator Motion
$e_{0}=$ Time-Varying Eccentricity of the Harmonic Oscillator Motion before Rendezvous
$e_{\text {ISS }}=$ International Space Station Orbit's Eccentricity
$\phi=$ Phase of Forcing Term in Out-of-plane Motion in Schweighart-Sedwick Equations
$E \quad=\quad$ Spacecraft Relative Mechanical Energy with respect to the Target (per Unit Mass)
$h=$ Target Altitude above the Earth Surface
$k=$ Coefficient in Schweighart-Sedwick Equations
$i_{\text {ref }}=$ Reference LVLH Orbit Inclination
$i_{\text {ISS }}=$ International Space Station Orbit's Inclination
ISS $=$ International Space Station
$J_{2}=$ Second Order Harmonic of Earth Gravitational Potential Field (Earth Flattening) (108263×10-8 [1])
LVLH $=$ Local Vertical Local Horizontal Cartesian Coordinates System
$l \quad=\quad$ Coefficient in Schweighart-Sedwick Equations (out-of-plane motion)
$m=$ Spacecraft Mass
$N_{S}=\quad$ Number of Spacecraft in the Fleet
$v=$ Earth Gravitational Constant (398600.4418 $\mathrm{km}^{3} \mathrm{~s}^{-2}$ [1])
$v_{\text {ISS }}=$ International Space Station Orbit's generic True Anomaly (at initial time)
$x, y, z=$ Position Coordinates of Target in LVLH
$\dot{x}, \dot{y}, \dot{z} \quad=\quad$ Velocity Components of Target in LVLH
$\omega=$ Target Circular Orbit Angular Velocity
$\omega_{\text {ISS }}=$ International Space Station Orbit's Argument of Perigee
$\Omega_{\text {ISS }}=$ International Space Station Orbit's Right Ascension of Ascending Node (RAAN)
$q=$ Coefficient in Schweighart-Sedwick Equations (out-of-plane motion)
$R_{\oplus}=$ Earth Mean Radius (6378.1363 km [1])
$r_{\text {ref }}=$ Reference LVLH Orbit Radius
$\rho \quad=\quad$ Atmospheric Density
$S=$ Spacecraft Wind-Cross Section Area
$s=$ Coefficient in Schweighart-Sedwick Equations
$T=$ Orbital Period
$t=$ Time
$t_{s_{i}}=i-$ th Sign Switching Instant for Differential Drag
$u, U=$ Control Variable
$V=$ Spacecraft Velocity Vector with respect to Earth Atmosphere
$\hat{V}=$ Spacecraft Velocity Unit Vector with respect to Earth Atmosphere

$z^{\prime}=\left[\begin{array}{llll}z_{1}^{\prime} & z_{2}^{\prime} & z_{3}^{\prime} & z_{4}^{\prime}\end{array}\right]^{T}=$ Intermediate Transformed Spacecraft Relative State Vector
$\mathbf{z}=\left[\begin{array}{llll}z_{1} & z_{2} & z_{3} & z_{4}\end{array}\right]^{T}=$ Transformed Spacecraft Relative State Vector
$(\ldots)_{0}=$ Initial Conditions
$(\ldots)_{j}=$ Component along $j$ direction in LVLH $(j=x, y, z)$

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## I. Introduction

THIS paper introduces a novel control method for autonomous orbit stabilization and rendezvous of a group of multiple spacecraft by using the differential aerodynamic drag. The proposed method can be used, for instance, for an on-orbit assembly mission. By varying the level of aerodynamic drag of each spacecraft, relative differential accelerations are generated among the spacecraft of the group, and, therefore, their relative orbits are controlled. The first obvious advantage of this technique is the propellant saving with respect to standard control by thrusters. A further advantage is avoiding thrusters' plumes impingement when spacecraft are close.

The use of aerodynamic drag to control low Earth orbiting spacecraft has been studied in [2] and [3] for the orbit control of a single spacecraft and in [4], [5], [6] and [7] for the formation-keeping and rendezvous of two spacecraft.

Most of the authors who focus on low thrust proximity maneuvers (see for instance [2-10]) use the classic Hill-Clohessy-Wiltshire linear model. 11 However, when the maneuver lasts for several orbits, a different representation of the relative dynamics is desired in order to take into account differential effects on the spacecraft motion due to the Earth flattening ( $\mathrm{J}_{2}$ perturbation).

The main contributions of the present work, which are original to the best of our knowledge, are:

1. We have significantly improved the method proposed by Leonard et al. ([6] and [7]) for the single chaser-single target rendezvous with no $\mathrm{J}_{2}$ effect, which we used as a starting point for our study. In particular, (1) we have eliminated the problem of having a residual distance between the chaser and the target at the end of the maneuver, (2) we have eliminated the need of using a numerical optimization routine, since the maneuver is based on an analytical expression.
2. We have developed a control policy based on aerodynamic drag for the rendezvous of a set of more than two spacecraft.
3. We have considered the presence of the $\mathrm{J}_{2}$ perturbation in the dynamic model used by the controller. In particular we used the model developed by Schweighart and Sedwick. ${ }^{12}$ Nevertheless the proposed approach can also be used with the simpler Hill-Clohessy-Wiltshire model.
The main difficulty when dealing with more than two spacecraft is to respect the existing constraints among the values of differential drag the spacecraft can experience. In fact, at a given time, some chaser spacecraft may need a drag force higher than the target's, while other chaser spacecraft may need a drag force equal or lower than the target's. These conflicts are here resolved through the introduction of a sequential logic based on the value of the relative mechanical energy of each chaser spacecraft with respect to the target.

The proposed control approach consists of the following two successive phases:

1. First, all of the chaser spacecraft (of arbitrary number $N_{s}$ ) are driven to closed relative orbits with respect to the target spacecraft. This stabilization is performed by simultaneously controlling the differential aerodynamic drag.
2. Second, additional closing maneuvers are performed, by each spacecraft at a time, in order to have all of the spacecraft converging to the target.
The paper is organized as it follows: Section II presents the dynamic model. Section III introduces the control algorithms. Section IV reports the results of the numerical simulations. Section V concludes the paper.

## II. Model of Relative Spacecraft Dynamics and Actively Controlled Differential Drag

The linearized dynamic model of a spacecraft moving with respect to a circular orbit, including the $\mathrm{J}_{2}$ effects, as introduced by Schweighart and Sedwick in [12], is

$$
\left\{\begin{array}{l}
\ddot{x}=2(\omega c) \dot{y}+\left(5 c^{2}-2\right) \omega^{2} x-3 \omega^{2} J_{2}\left(R_{\otimes}^{2} / r_{r e f}\right) .  \tag{1}\\
\quad \cdot\left\{\frac{1}{2}-\left[3 \sin ^{2} i_{r e f} \sin ^{2}(k t) / 2\right]-\left[\left(1+3 \cos 2 i_{r e f}\right) / 8\right]\right\}+U_{x} ; \quad k=\omega c+\frac{3 \omega J_{2} R_{\otimes}^{2}}{2 i_{r e f}^{2}} \cos ^{2} i_{r e f} \\
\ddot{y}=-2(\omega c) \dot{x}-3 \omega^{2} J_{2}\left(R_{\otimes}^{2} / r_{r e f}\right) \sin ^{2} i_{r e f} \sin (k t) \cos (k t)+U_{y} \\
\ddot{z}=-q^{2} z+2 l q \cos (q t+\phi)+U_{z}
\end{array}\right.
$$

where the coordinate system is thus defined: the $X$ axis points from the center of the Earth to the origin of the system (which moves along a circular orbit), the $y$ axis is along the orbital track and the $Z$ axis completes a righthand Cartesian coordinate system. The angular velocity of the coordinate system with respect to the inertial frame is ${ }^{12}$

$$
\begin{equation*}
\omega c, \quad c=\sqrt{1+\frac{3 J_{2} R_{\oplus}^{2}}{8 r_{r e f}^{2}}\left(1+3 \cos 2 i_{r e f}\right)} \tag{2}
\end{equation*}
$$

In the present paper, one of the spacecraft, in a fleet of multiple spacecraft, is arbitrarily chosen as the target and all of the other spacecraft need to maneuver to reach it. Therefore, of particular interest for our purposes are the following equations describing the relative dynamics between a generic spacecraft of the fleet and the target ${ }^{12}$

$$
\left\{\begin{array}{l}
\Delta \ddot{x}=2(\omega c) \Delta \dot{y}+\left(5 c^{2}-2\right) \omega^{2} \Delta x+u_{x}  \tag{3}\\
\Delta \ddot{y}=-2(\omega c) \Delta \dot{x}+u_{y} \\
\Delta \ddot{z}=-q^{2} \Delta z+2 l q \cos (q t+\phi)+u_{z}
\end{array}\right.
$$

where $\left[\begin{array}{lll}u_{x} & u_{y} & u_{z}\end{array}\right]$ indicate the components of the relative acceleration between the two spacecraft due to a control action.

Eq. (1) and Eq. (3) reduce to the Hill-Clohessy-Wiltshire equations if the $\mathrm{J}_{2}$ effect is not considered, i.e. when $c=1$ and $l=q=0$ (see [12]). Therefore, all of the developments of the present work remain valid if the simpler Hill-Clohessy-Wiltshire model is used.

The acceleration on a spacecraft due to the atmospheric drag can be expressed as ${ }^{6,7}$

$$
\begin{equation*}
\vec{a}=\left(-\frac{\rho S C_{D}}{2 m} V^{2}\right) \hat{V} \tag{4}
\end{equation*}
$$

For spacecraft able to change their wind-cross sectional surface area $S$, this acceleration can be considered as a control vector, with the only non-zero component $u_{y}$ in Eq. (3). The simplifying assumption of controlling only along $y$ can be found in several works, not only limited to drag control techniques ([8], [13] and [14]). A controlled variation of the wind-cross surface $S$ can be achieved, for instance, by changing the attitude of the spacecraft or, as considered in this paper, by opening and closing a drag plate.

We consider the following modeling assumptions for our study (see Figure 1):

1. The angle of attack of the drag plate of each spacecraft can be either 0 or 90 degrees, thus generating either a minimum or a maximum drag force (off-on) with no intermediate values considered possible.
2. Attitude dynamics is not considered. Attitude is assumed to be stabilized.
3. All of the spacecraft in the fleet have the same drag coefficient and mass.
4. The air density is constant for all of the spacecraft and equal to that of the target's altitude at the initial time $t_{0}$.
5. The problem is confined to the $x y$ plane. Therefore, for each chaser, the state vector is $\left[\begin{array}{llll}\Delta x & \Delta y & \Delta \dot{x} & \Delta \dot{y}\end{array}\right]^{T}$ and the final condition is $\left[\begin{array}{llll}\Delta x & \Delta y & \Delta \dot{x} & \Delta \dot{y}\end{array}\right]^{T}=\left[\begin{array}{llll}0 & 0 & 0 & 0\end{array}\right]^{T}$. The control of the dynamics along the z axis, which is oscillatory and independent from the one on the xy plane, is considered beyond of the scope of the present paper.
6. The target orbital rate $\omega$ is constant during the maneuver.


Figure 1. Conceptual explanation of the differential drag control. If the chaser opens its drag plates it causes a relative negative acceleration of the chaser with respect to the target; if the target opens its drag plates it causes a relative positive acceleration of the chaser with respect to the target.

## A. State Vector Transformation and Phase Planes

This section summarizes the results of [15], where we applied a state vector transformation to Eq. (3) in order to separate the mean secular motion from the oscillatory part. The generic chaser's state vector $\left[\begin{array}{llll}\Delta x & \Delta y & \Delta \dot{x} & \Delta \dot{y}\end{array}\right]^{T}$ of the Schweighart-Sedwick equations is transformed into a new intermediate state vector $\left[\begin{array}{llll}z_{1}^{\prime} & z_{2}^{\prime} & z_{3}^{\prime} & z_{4}^{\prime}\end{array}\right]^{T}$ by

$$
\left[\begin{array}{l}
z_{1}^{\prime}  \tag{5.a}\\
z_{2}^{\prime} \\
z_{3}^{\prime} \\
z_{4}^{\prime}
\end{array}\right]=\left[\begin{array}{cccc}
0 & 1 & -\frac{A}{A^{2}-B} & 0 \\
-\frac{B A}{A^{2}-B} & 0 & 0 & -\frac{B}{A^{2}-B} \\
0 & 0 & -\frac{A^{2}}{2\left(A^{2}-B\right)^{3 / 2}} & 0 \\
-\frac{A^{2} B}{2\left(A^{2}-B\right)^{3 / 2}} & 0 & 0 & -\frac{A^{3}}{2\left(A^{2}-B\right)^{3 / 2}}
\end{array}\right]\left[\begin{array}{c}
\Delta x \\
\Delta y \\
\Delta \dot{x} \\
\Delta \dot{y}
\end{array}\right] ; \quad A=2(\omega c), B=\left(5 c^{2}-2\right) \omega^{2}
$$

where $A^{2}-B$ is an always positive constant.
Eq. (5.b) reports the inverse transformation of Eq. (5.a)

$$
\left[\begin{array}{c}
\Delta x  \tag{5.b}\\
\Delta y \\
\Delta \dot{x} \\
\Delta \dot{y}
\end{array}\right]=\left[\begin{array}{cccc}
0 & -\frac{A}{B} & 0 & -\frac{2 \sqrt{A^{2}-B}}{A^{2}} \\
1 & 0 & \frac{2 \sqrt{A^{2}-B}}{A} & 0 \\
0 & 0 & -\frac{2 \sqrt{\left(A^{2}-B\right)^{3}}}{A^{2}} & 0 \\
0 & 1 & 0 & \frac{2 \sqrt{A^{2}-B}}{A}
\end{array}\right]\left[\begin{array}{l}
z_{1}^{\prime} \\
z_{2}^{\prime} \\
z_{3}^{\prime} \\
z_{4}^{\prime}
\end{array}\right]
$$

By expressing $\left[\begin{array}{llll}\Delta x & \Delta y & \Delta \dot{x} & \Delta \dot{y}\end{array}\right]^{T}$ as a function of $\left[\begin{array}{llll}z_{1}^{\prime} & z_{2}^{\prime} & z_{3}^{\prime} & z_{4}^{\prime}\end{array}\right]^{T}$ (Eq. (5.b)), substituting $\left[\begin{array}{llll}\Delta x & \Delta y & \Delta \dot{x} & \Delta \dot{y}\end{array}\right]^{T}$ into Eq. (3), and considering only a control component along $y$, it follows that

$$
\left[\begin{array}{c}
\dot{z}_{1}^{\prime}  \tag{6}\\
\dot{z}_{2}^{\prime} \\
\dot{z}_{3}^{\prime} \\
\dot{z}_{4}^{\prime}
\end{array}\right]=\left[\begin{array}{cccc}
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & -\left(A^{2}-B\right) & 0
\end{array}\right]\left[\begin{array}{c}
z_{1}^{\prime} \\
z_{2}^{\prime} \\
z_{3}^{\prime} \\
z_{4}^{\prime}
\end{array}\right]+\left[\begin{array}{c}
0 \\
-\frac{B}{A^{2}-B} \\
0 \\
\frac{A^{3}}{2\left(A^{2}-B\right)^{3 / 2}}
\end{array}\right] u_{y}
$$

which is the combination of a double integrator and a harmonic oscillator. Eq. (6) has the following closed form solution for constant control acceleration $u_{y}$

$$
\begin{align*}
& z_{1}^{\prime}=-\frac{B}{A^{2}-B} u_{y} \frac{t^{2}}{2}+z_{2_{0}}^{\prime} t+z_{1_{0}}^{\prime} \\
& z_{2}^{\prime}=-\frac{B}{A^{2}-B} u_{y} t+z_{z_{0}}^{\prime} \\
& z_{3}^{\prime}=\left(z_{3_{0}}^{\prime}-\frac{A^{3} u_{y}}{2\left(A^{2}-B\right)^{5 / 2}}\right) \cos \left[\left(\sqrt{A^{2}-B}\right) t\right]+\frac{z_{4_{0}}^{\prime}}{\sqrt{A^{2}-B}} \sin \left[\left(\sqrt{A^{2}-B}\right) t\right]+\frac{A^{3} u_{y}}{2\left(A^{2}-B\right)^{5 / 2}}  \tag{7.a}\\
& z_{4}^{\prime}=z_{4_{0}}^{\prime} \cos \left[\left(\sqrt{A^{2}-B}\right) t\right]-\sqrt{A^{2}-B}\left(z_{3_{0}}^{\prime}-\frac{A^{3} u_{y}}{2\left(A^{2}-B\right)^{5 / 2}}\right) \sin \left[\left(\sqrt{A^{2}-B}\right) t\right]
\end{align*}
$$

For notation convenience we will use, for the rest of the paper, the following modified final transformed state vector

$$
\begin{align*}
& z_{1}=z_{1}^{\prime} \\
& z_{2}=z_{2}^{\prime} \\
& z_{3}=z_{3}^{\prime}  \tag{7.b}\\
& z_{4}=\frac{z_{4}^{\prime}}{\sqrt{A^{2}-B}}
\end{align*}
$$

By eliminating the time variable from the first two equations of Eq. (7) it follows that the motion in the $z_{1} z_{2}$ plane occurs along parabolas with positive or negative concavity depending on the sign of the differential drag (see Figure 2.a).

Furthermore, by eliminating the time variable from the last two equations of Eq. (7), it follows that the motion in the $z_{3} Z_{4}$ plane occurs along circles centered at the points $\left[\frac{A^{3} u_{y}}{2\left(A^{2}-B\right)^{5 / 2}} 0\right]$, with either $u_{y}>0$ or $u_{y}<0$ (see

## Figure 2.b).

As it can be easily demonstrated from Eq. (7), the uncontrolled trajectories (coasting) correspond, in the plane $z_{1} z_{2}$, to horizontal straight lines traveled with a direction dependent on the sign of the initial $z_{2}$, and, in the plane $z_{3} z_{4}$, to circles centered at the origin with a radius equal to the initial distance from the origin and traveled in the counterclockwise direction.

All of the state variables (in both the phase planes) need to be controlled in order to move toward the desired final rendezvous condition of zero relative position and velocity.

The bold curves in Figure 2.a are the switching curves taking the average position of the chaser with respect to the target directly to the origin without additional need of switching the sign.

a) Qualitative shape of the curves in $Z_{1} z_{2}$

b) Qualitative shape of the curves in $Z_{3} Z_{4}$

Figure 2. Qualitative shape of the curves representing the relative motion of a chaser with respect to the target in the phase planes. The axis orientation has been chosen consistent with [6].

The distance from the origin of a point on the curves in Figure 2.b is given, at any time, by

$$
\begin{equation*}
e=\sqrt{z_{3}^{2}+z_{4}^{2}} \tag{8}
\end{equation*}
$$

This quantity will be called the eccentricity of the harmonic motion. ${ }^{6}$

## III. Multi-Spacecraft Control Algorithm

The aim of the proposed maneuver, thought to be in preparation of an eventual assembly, is to drive the state of each chaser to the origin of both phase planes in Figure 2. The maneuver is conducted in the following two successive phases:

1. Stabilization phase: each chaser spacecraft is driven to a stable periodic orbit around the target.
2. Rendezvous phase: each chaser spacecraft converges to the target.

## A. Two spacecraft case (one chaser, one target)

Let us consider first, for explanation purposes, to have only two spacecraft in the fleet (one chaser and one target). A negative relative acceleration along the $y$ axis of the chaser with respect to the target due to the drag appears if the chaser is opening the plate while the target is not. Conversely, a positive acceleration of the chaser with respect to the target appears if the target is opening the plate while the chaser is not. No relative acceleration appears when both vehicles have their plates either open or closed.

## 1. Phase one: Relative Orbit Stabilization

The conditions to be satisfied in order to have closed relative orbits, as can be demonstrated from Eq. 3, are ${ }^{12}$

$$
\begin{equation*}
\Delta \dot{x}=\frac{\omega \Delta y}{2} \frac{(1-s)}{\sqrt{1+s}}, \quad \Delta \dot{y}=-2 \omega \Delta x \sqrt{1+s} ; \quad s=\frac{3 J_{2} R_{\oplus}^{2}}{8 R^{2}}\left(1+3 \cos 2 i_{r e f}\right) \tag{9}
\end{equation*}
$$

A particular case of this condition is when $\Delta x=\Delta \dot{x}=0$, i.e. for a leader-follower configuration, which is a particular admissible case of stable relative orbit. Eq. (9) corresponds to $z_{1}=z_{2}=0$, as it can be demonstrated by equating to zero the first two equations of Eq. (5).

Notably, if the differential drag has the opposite sign of $z_{4}$, the value of $e$ reduces with time. In fact, by taking the derivative with respect to time of Eq. (8), and by considering Eq. (7), it follows that

$$
\begin{equation*}
\frac{d\left(e^{2}\right)}{d t}=2 \omega \frac{A^{3} u_{y}}{2\left(A^{2}-B\right)^{5 / 2}} z_{4} \tag{10}
\end{equation*}
$$

The control sequence has to take into account the conditions related to each of the two phase planes at the same time. In fact, if we consider only the double integrator portion of the dynamics (see Figure 2.a), by switching the sign of the relative acceleration just once, at the point where one of the two switching curves is reached, the spacecraft state variables $\left(z_{1}, z_{2}\right)$ would go to the origin in a minimum time. ${ }^{16}$ But this control procedure could result in a high residual eccentricity $e$ (see Figure 3.b and Eqq. (5) and (6)).

On the other hand, if we consider only the harmonic oscillator portion of the dynamics (see Figure 2.b), by suitably switching the sign of the relative acceleration, as dictated by the optimal control theory, the spacecraft state variables $\left(z_{3}, z_{4}\right)$ go to the origin in a minimum time (see [16]). But this control procedure would result, in general, in a residual drift of the chaser with respect to the target.

Therefore, in order to simultaneously control the four state variables $\left(z_{1}, z_{2}, z_{3}, z_{4}\right)$ on the two phase planes of Figure 2, the following algorithmic steps are applied (see also Figure 3):

1. If, at the beginning of the maneuver, the point $\left(z_{1}, z_{2}\right)$ is either in the A quadrant of Figure 2.a at the left hand side of the switching curve or in the $C$ quadrant at the right hand side of the switching curve, then go directly to Step 2 . Conversely, if at the beginning of the maneuver the point $\left(z_{1}, z_{2}\right)$ is either on the right hand side of the control switching curve for $z_{2}<0$ or on the left hand size of the switching curve for $z_{2}>0$ (see Figure 2.a), then either a negative or a positive relative acceleration is applied, respectively, until either the C quadrant or the A quadrant is reached. Then go to Step 2.
2. Either a positive relative acceleration is applied if the point $\left(z_{1}, z_{2}\right)$ is in the A quadrant of Figure 2.a, or a negative acceleration if the point is in the C quadrant, until any one of the following three conditions is met:
a. The sign of the variable $z_{4}$ switches. Then go to Step 3.
b. The condition $e<\frac{A^{3} u_{y}}{\left(A^{2}-B\right)^{5 / 2}}$ becomes true. Then go to Step 4.
c. One of the two switching curves of Figure 2.a is reached. Then go to Step 5.
3. The sign of the relative acceleration is switched and is then kept constant until any one of the three conditions considered during Step 2 is met again. Then the indications reported after the met conditions are followed.
4. Either a positive relative acceleration is applied if the point $\left(z_{1}, z_{2}\right)$ is in the A quadrant of Figure 2.a, or a negative acceleration if the point is in the C quadrant, until one of the two switching curves of Figure 2.a is reached. Go to Step 5.
5. The sign of the relative acceleration is switched for the last time. When the point $\left(z_{1}, z_{2}\right)$ reaches the origin, the sign of the relative acceleration is switched off, and the stabilization algorithm stops.
The quadrant A and C of Figure 2.a are called "saw-tooth" zones due to the shape of the curve resulting from the application of the above algorithm on the phase plane $z_{1} z_{2}$ (see Figure 3).

Notably, at any time during the iterative repetition of the steps 3-2.a, the future sign switches of $z_{4}$ are given, based on the current values of $\left(z_{3}, z_{4}\right)$, by

$$
\begin{equation*}
t_{s_{k}}=\frac{1}{\sqrt{A^{2}-B}}\left[\tan ^{-1}\left(\frac{z_{4}}{\left(z_{3}-\frac{A^{3} u_{y}}{2\left(A^{2}-B\right)^{5 / 2}}\right)}\right)+k \pi\right] \tag{11}
\end{equation*}
$$

as it can be demonstrated by equating to zero the last equation of Eq. (7).
Furthermore, the conditional step 2.b is based on the fact that there is no benefit, as far as eccentricity reduction, in switching the control sign according to the sign of $z_{4}$ (every half orbital period) when $e<\frac{A^{3} u_{y}}{\left(A^{2}-B\right)^{5 / 2}}$.


Figure 3. Qualitative example of relative orbit stabilization maneuver in the phase planes. The plus symbol $(+)$ on the trajectory indicates the initial state, the star symbol $\left(^{*}\right)$ indicates the final condition at the exit of the stabilization algorithm. Equal Greek letters in the two figures indicate simultaneous events.
2. Phase two: Rendezvous to the Target

This section introduces a new method, exploiting the differential drag, to drive to zero the relative state vector of a single chaser spacecraft with respect to the target, i.e. to reach the condition $z_{1}=z_{2}=z_{3}=z_{4}=0$, once the condition of stable relative orbit ( $z_{1}=z_{2}=0$ ) has been reached as described in the previous section.

In particular, the trajectory in the $z_{1} z_{2}$ plane, during the rendezvous maneuver, is assumed to be one element of the set $S_{12}$ of infinite trajectories which start from the origin of the phase plane and are composed of the following three controlled phases (see also Figure 4.a):

1. A maximum (or minimum) acceleration phase of arbitrary time duration $\Delta t^{*}$;
2. A minimum (or maximum) acceleration phase of time duration $2 \Delta t^{*}$;
3. A maximum (or minimum) acceleration phase of time duration $\Delta t^{*}$.

These trajectories are closed and symmetric with respect to the $z_{1}$ axis.
Furthermore, the trajectory in the $z_{3} z_{4}$ plane, during the same maneuver, is assumed to be one element of the set $S_{34}$ of $\infty^{2}$ trajectories which start from the state at the end of the relative orbit stabilization and are composed of:

1. An initial coasting phase of arbitrary duration $\Delta t_{w}$;
2. A sequence of three controlled phases of total duration $4 \Delta t^{*}$, corresponding to the phases of the trajectories on the $z_{1} z_{2}$ plane which are part of the set $S_{12}$.
The specific values of $\Delta t^{*}$ and $\Delta t_{w}$, and of the acceleration sign sequence, are chosen to be the ones which identify, among the elements of the set $S_{34}$, the shortest (in time) trajectory which connects the state at the end of the stabilization phase ( $\left[\bar{z}_{3} \bar{z}_{4}\right]$ ) to the desired final rendezvous condition ( $z_{3}=z_{4}=0$ ).

In detail, the following steps were followed in order to obtain the analytic solution for the value of $\Delta t^{*}$ (see also Figure 4):

1. From Eq. (7), considering the desired rendezvous ( $z_{3}=z_{4}=0$ ) as initial condition, the analytic symbolic expression is obtained for the state reached in the $z_{3} z_{4}$ phase plane by applying either the maximum or the minimum acceleration for an arbitrary backward time interval $-\Delta t^{*}$. The decision of which sign of acceleration to use will become clear later. Let us use the expression
$\left[{ }^{+} z_{3}\left(-\Delta t^{*}\right){ }^{+} z_{4}\left(-\Delta t^{*}\right)\right]$ in order to indicate the state reached with maximum acceleration, and the expression $\left[{ }^{-} z_{3}\left(-\Delta t^{*}\right){ }^{-} z_{4}\left(-\Delta t^{*}\right)\right]$ in order to indicate the state reached with minimum acceleration. Furthermore, for nomenclature convenience, let us use the expression $\left[{ }^{ \pm} z_{3}\left(-\Delta t^{*}\right){ }^{ \pm} Z_{4}\left(-\Delta t^{*}\right)\right]$ in order to shortly indicate both cases.
2. From Eq. (7), considering as initial condition the one achieved at the end of step one, the analytic symbolic expression is obtained for the state reached by applying the extremal acceleration of opposite sign with respect to that of step one for a backward time interval $-2 \Delta t^{*}$. The two possible reached states are indicated by the expression $\left[{ }^{ \pm} Z_{3}\left(-3 \Delta t^{*}\right){ }^{ \pm} Z_{4}\left(-3 \Delta t^{*}\right)\right]$.
3. From Eq. (7), considering as initial condition the one achieved at the end of step two, the analytic symbolic expression is obtained for the state reached by applying the extremal acceleration of same sign with respect to that of step one for a backward time interval $-\Delta t^{*}$. The two possible reached states are indicated by the expression $\left[{ }^{ \pm} z_{3}\left(-4 \Delta t^{*}\right){ }^{ \pm} z_{4}\left(-4 \Delta t^{*}\right)\right]$.
4. The following analytic symbolic expression is found for the distance between the state obtained in the $Z_{3} z_{4}$ plane at the end of step three and the circle of radius $e_{0}$, which is the orbit reached at the end of the stabilization phase:

$$
\begin{equation*}
{ }^{ \pm} d\left(\Delta t^{*}\right)=e_{0}-\sqrt{\left({ }^{ \pm} Z_{3}\left(-4 \Delta t^{*}\right)\right)^{2}+\left({ }^{ \pm} Z_{4}\left(-4 \Delta t^{*}\right)\right)^{2}} \tag{12}
\end{equation*}
$$

5. By equating to zero the expression in Eq. 12, the following equation in the unknown $\Delta t^{*}$ is found, which is independent from the particular acceleration sign sequence used:

$$
\begin{align*}
& { }^{+} d\left(\Delta t^{*}\right)=^{-} d\left(\Delta t^{*}\right)=e_{0}+K \sqrt{\left(-5+4 \cos \left(f \Delta t^{*}\right)-4 \cos \left(3 f \Delta t^{*}\right)+\cos \left(4 f \Delta t^{*}\right)+4 \cos \left(2 f \Delta t^{*}\right)\right)}=0  \tag{13}\\
& f=\sqrt{A^{2}-B}, \quad K=-\frac{\sqrt{2}}{2} \frac{A^{3}\left|u_{y}\right|}{f^{5}} i
\end{align*}
$$

where $i$ is the imaginary unit. Four solutions of Eq. 13 exist, of which two are real and two are complex conjugates. The smallest one between the two real solutions is chosen as the required $\Delta t^{*}$. This solution is:

$$
\begin{align*}
& \Delta t^{*}=\frac{1}{f} \cos ^{-1}\left(1 / 2+(1 / 12) \sqrt{36+6 \sqrt[3]{H} / K-\frac{36 e_{0}^{2}}{K \sqrt[3]{H}}}-(\sqrt{6} / 12) \sqrt{12-\sqrt[3]{H} / K+\frac{6 e_{0}^{2} / K \sqrt[3]{H}-72}{} / \sqrt{36+6 \sqrt[3]{H} / K-\frac{36 e_{0}^{2} / K \sqrt[3]{H}}{}}}\right)  \tag{14}\\
& H=-54 K e_{0}^{2}+6 \sqrt{3} e_{0}^{2} \sqrt{2 e_{0}^{2}+27 K^{2}}
\end{align*}
$$

In summary, this $\Delta t^{*}$ guarantees to reach the rendezvous condition by following a maneuver consisting of three bang-bang controlled periods (of respective duration $\Delta t^{*}$, $2 \Delta t^{*}$ and $\Delta t^{*}$ ) starting from a state $\left[{ }^{ \pm} z_{3}\left(-4 \Delta t^{*}\right){ }^{ \pm} z_{4}\left(-4 \Delta t^{*}\right)\right]$ along the stable orbit reached at the end of the stabilization phase.

The value of the duration $\Delta t_{w}$ of the initial coasting along that stable orbit is straightforwardly found by solving the following equation:

$$
\begin{equation*}
\Delta t_{w}=\min \left(\left|\frac{{ }^{ \pm} \Delta \lambda_{\text {wait }}}{f}\right|\right)=\frac{1}{f} \min \left(\left|\tan ^{-1}\left(\frac{{ }^{ \pm} z_{3}\left(-4 \Delta t^{*}\right)}{{ }^{ \pm} z_{4}\left(-4 \Delta t^{*}\right)}\right)-\tan ^{-1}\left(\frac{\bar{Z}_{3}}{\bar{Z}_{4}}\right)\right|\right) \tag{15}
\end{equation*}
$$

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Finally, the acceleration sign sequence is chosen, among the two possible ones, which corresponds to the duration $\Delta t_{w}$.

The solution proposed in this section presents the following significant advantages with respect to the one proposed in [6]:

1. The rendezvous control policy developed here drives the whole state vector of the chaser to zero, while the procedure in [6] always results in a final residual distance of the chaser from the target.
2. It is an analytical solution on a single variable, while a numerical optimization solution is required in [6], where two different time durations have to be computed.
3. The time required for the maneuver is much lower, as it is shown in the following numerical simulations.


Figure 4. Qualitative example of rendezvous maneuver in the phase planes. In figure b the plus symbol (+) indicates the initial state, and the ex symbol ( x ) indicates the final condition.

## B. Multiple spacecraft case ( $\boldsymbol{N}_{s}-1$ chasers, one target; with $\boldsymbol{N}_{s}>2$ )

## 1. Phase one: Relative Orbit Stabilization of Multiple Spacecraft

A centralized heuristic control logic is here proposed to stabilize the orbits of multiple chaser spacecraft about the target by exploiting the differential drag. When multiple chaser spacecraft are involved, the problem becomes more challenging as the achievable levels of differential drag between each chaser and the target are mutually constrained. Consider, as an example, the case when some of the chasers' orbits around the target are already stabilized while others are not: if the stabilization of one of the remaining unstable chasers requires the target to experience maximum drag, the already stabilized chasers have to open their plates as well for the sake of keeping their relative stabilization.

In particular, the proposed control logic is composed of the following algorithmic steps. These steps are iteratively executed till the desired condition of orbit stabilization is reached for all of the chaser spacecraft (i.e. until it is $z_{1}=z_{2}=0$ for each chaser with respect to the target):

1. The required relative acceleration is computed, at the beginning of each sample time interval $\Delta t_{s}$, for each of the chaser spacecraft that remains to be stabilized around the target. The control algorithm described in Section III A. 1 for a single chaser spacecraft is used to compute the required acceleration.
2. If the required relative accelerations computed in step 1 are all in accordance with each other, they are commanded (and generated by synchronously maneuvering the drag plates of all of those chasers). Otherwise, if not all of the required relative accelerations computed in step 1 are in accordance with each other, then the level of relative acceleration applied to all of the chasers that remain to be stabilized around the target is decided based on the following prioritized list of conditions:
A. When one of the not yet stabilized chaser spacecraft reaches a switching curve (Figure2.a), the relative acceleration which brings that spacecraft to a stable orbit (i.e. to the condition $z_{1}=z_{2}=0$ ) is considered as the reference acceleration till the stabilization is achieved (for that single spacecraft). Those chasers requiring an opposite sign acceleration from step 1 experience zero control (coasting); the others will experience the reference acceleration.
B. When one of the not yet stabilized chaser spacecraft meets the condition 2.b of Section III A.1, its desired relative acceleration (according to algorithm in Section III A.1) is considered as the reference acceleration till the stabilization is achieved. Those chasers requiring an opposite sign acceleration from step 1 experience zero control (coasting); the others will experience the reference acceleration. The occurrence of condition 2.A above would cause immediate jump to the case 2.A.
C. When one of the not yet stabilized chaser spacecraft crosses the $z_{1}$ axis (Figure 2.a) and enters into one of the saw-tooth zones, its desired relative acceleration value is considered as the reference acceleration till one of the conditions 2.a, 2.b, or 2.c of Section III A. 1 is verified. Those chasers requiring an opposite sign acceleration from step 1 experience zero control (coasting), the others will experience the reference acceleration.
D. When none of the three conditions above are satisfied, a reference relative acceleration value is computed for all the not yet stabilized chasers, which is equal to the acceleration desired by the chaser spacecraft which has the highest relative energy with respect to the target, as determined by

$$
\begin{equation*}
E=v \sqrt[3]{R}|x|+\dot{x}^{2}+\dot{y}^{2}=v \sqrt[3]{R}\left|-\frac{A^{3} z_{2}+2 B z_{4}}{A^{2} B}\right|+\left(\frac{2 \sqrt{A^{2}-B} z_{3}}{A^{2}}\right)^{2}+\left(\frac{A z_{2}+2 z_{4}}{A}\right)^{2} \tag{16}
\end{equation*}
$$

Those chasers requiring an opposite sign acceleration from step 1 experience zero control (coasting); the others will experience the reference acceleration.
An energy dead-band $\Delta E$ is used in order to avoid chattering among different values of acceleration when the energies of different spacecraft become comparable. The occurrence of any of the conditions 2.A, 2.B or 2.C will cause immediate jump to the corresponding case.

## 2. Phase two: Rendezvous of Each Chaser Spacecraft to the Target

Once that every chaser spacecraft has been stabilized about the target, the rendezvous with the target of each chaser at a time is sequentially performed by following the algorithm introduced for the two spacecraft case in Section III A.2. In particular the chaser to maneuver first is the one which has the smallest relative orbit about the target. Therefore, if the relative orbits do not have any intersection, collisions are impossible.

## 3. Collision Avoidance

Potential collisions are possible during all of the phases of the maneuver and need to be taken into account. Because the control calculation completely analytical, the full maneuver sequence can be recomputed with a small computational burden. Therefore the following simple strategy is adopted for collision avoidance: the whole maneuver sequence is pre-computed and, if any collisions are foreseen, an initial coasting time is iteratively introduced for the whole fleet until all collisions are avoided.

## IV. Simulation Results

This section reports the simulation results for the sample case of a fleet of five homogenous spacecraft ( 1 target and 4 chasers). The proposed control policy is applied first by considering the linear dynamics model. In order to further validate the reliability and robustness of the proposed method, an application of the algorithm to a nonKeplerian orbital propagator is also presented.

## A. Linear Dynamics Simulation (5 spacecraft)

Table 1 reports the chosen values for the main characteristics of the spacecraft, Table 2 reports the values of the initial conditions, and Table 3 reports the values of additional simulation parameters.

Table 1: Spacecraft Characteristics and Atmospheric Density

| Table 1: Spacecraft Characteristics and Atmospheric Density |  |
| :---: | :---: |
| Mass $(\mathrm{kg})$ | 10 |
| $S\left(\mathrm{~m}^{2}\right)$ | 1 |
| $C_{D}$ | 2.2 |
| $h(\mathrm{~km})$ | 350 |
| $\rho_{350 \mathrm{~km}}\left(\mathrm{~kg} \cdot \mathrm{~m}^{-3}\right)$ | $6.98 \cdot 10^{-12}([17])$ |
| $i_{\text {ref }}=i_{\text {ISS }}(\mathrm{deg})$ | 51.595 |

Table 2: Initial Conditions for Simulation

|  | $Z_{1_{0}}(m)$ | $z_{2_{0}}(m / s)$ | $z_{3_{0}}(m)$ | $z_{4_{0}}(m)$ | $\left[\begin{array}{ll}x_{0} & y_{0}\end{array}\right](m)$ | $\left[\begin{array}{ll}\dot{x}_{0} & \dot{y}_{0}\end{array}\right](\mathrm{m} / \mathrm{s})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sat $_{1}$ | -609 | 0.86 | 128 | 43.71 | $\left[\begin{array}{ll}-528.38 & -481\end{array}\right]$ | $\left[\begin{array}{ll}0.07 & 0.92\end{array}\right]$ |
| Sat $_{2}$ | -609 | -0.86 | 128 | -43.71 | $\left[\begin{array}{ll}528.38 & -481\end{array}\right]$ | $\left[\begin{array}{ll}0.07 & -0.92\end{array}\right]$ |
| Sat $_{3}$ | -509 | -0.69 | 128 | 52.45 | $\left[\begin{array}{ll}379.62 & -381\end{array}\right]$ | $\left[\begin{array}{ll}0.07 & -0.64\end{array}\right]$ |
| Sat $_{4}$ | 309 | -0.52 | 128 | 52.45 | $\left[\begin{array}{ll}279.62 & 437\end{array}\right]$ | $\left[\begin{array}{ll}0.07 & -0.47\end{array}\right]$ |
| Sat $_{5}$ (target) | 0 | 0 | 0 | 0 | $\left[\begin{array}{ll}0 & 0\end{array}\right]$ | $\left[\begin{array}{ll}0 & 0\end{array}\right]$ |

Table 3: Simulation Parameters

| $\Delta t_{s}$ | 10 s |
| :---: | :---: |
| $\Delta E$ | Dynamic: half of the current maximum energy |
| $d_{c}$ | 20 m |

The maximum value of drag for any spacecraft of the fleet (drag plate opened orthogonally to the velocity direction) corresponding to the parameter values listed in Table 1 is

$$
\begin{equation*}
u_{y}=4.55 \cdot 10^{-5} \mathrm{~m} \cdot \mathrm{~s}^{-2} \tag{17}
\end{equation*}
$$

Figures 5 to 11 report the simulation’s results corresponding to the value of parameters listed in Tables 1 to 3. The control sequence was determined according to the algorithms described in Section III. In particular, Figure 5 reports the trajectories of the spacecraft in the plane $z_{1} z_{2}$ during the stabilization phase of the maneuver. The control algorithm drives first the chaser spacecraft (designated as Sat ${ }_{4}$ ) into a stable orbit about the target (see Figure 5.a), then $\mathrm{Sat}_{3}$ (see Figure 5.b), $\mathrm{Sat}_{2}$ (see Figure 5.c) and $\mathrm{Sat}_{1}$ (see Figure 5.d), respectively. In the plots, crosses indicate initial states, stars final states (either referred to a single sequence or the overall stabilization phase). Portions of maneuver during which some of the spacecraft drift away can be recognized as straight lines parallel to the $z_{1}$ axis. Drifting phases arise, according to the algorithm introduced in Section III, when a spacecraft has to wait for the others which need to maneuver first due to their higher relative energy (Eq. (16)). Collisions did not occur during this simulation, i.e. no collision avoidance strategy was required.

a) First sequence: Sat $_{4}$ is stabilized

c) Third sequence: Sat $_{2}$ is stabilized

b) Second sequence: Sat $_{3}$ is stabilized

d) Fourth sequence: Sat $_{1}$ is stabilized

Figure 5. Simulation results: phase one, relative orbit stabilization, spacecraft trajectories in the $z_{1} z_{2}$ plane.
Figure 6 reports the evolution of the states $z_{3}$ and $z_{4}$ during the stabilization phase of the maneuver.



Figure 6. Simulation results: phase one, relative orbit stabilization, spacecraft trajectories in the $z_{3} z_{4}$ plane.
Figure 7 presents the time history of the differential drag control acting on each of the four chaser spacecraft during the four sequences of the stabilization maneuver corresponding to Figure 5. Positive drag indicates that a chaser is opening the drag plate while the target is not, negative indicates the target opening its drag plate while the chaser is not, and zero drag corresponds to one of the two following cases: a) both target and chaser are opening their plates; b) target and chaser are both keeping their plates closed. The control sequences in Figure 7 can be realistically implemented. In particular any chattering behavior is absent.

a) First sequence: Sat $_{4}$ is stabilized

b) Second sequence: Sat $_{3}$ is stabilized


Figure 7. Simulation results: phase one, relative orbit stabilization, differential drag acting on each chaser spacecraft with respect to the target during the four sequences corresponding to figure 5.

Figure 8 shows the trajectories in the $\Delta x \Delta y$ plane during the spacecraft stabilization phase.


Figure 8. Simulation results: phase one, relative orbit stabilization, spacecraft trajectories in the $\Delta x \Delta y$ plane. Stars indicate the beginning of the stable orbiting condition of each chaser about the target.

The required time for the stabilization phase of the maneuver was 12.04 hours ( 7.89 orbital periods). The residual distances of the chaser spacecraft from the target at the end of the stabilization phase are reported in Table 4.

Table 4: Stabilization Maneuvers Final Distances

| Sat $_{1}$ | 103.97 m |
| :--- | :--- |
| Sat $_{2}$ | 142.72 m |
| Sat $_{3}$ | 329.77 m |
| Sat $_{4}$ | 157.81 m |

After the stabilization phase, the final rendezvous is achieved by applying sequentially, to each of the chaser spacecraft, the algorithm introduce in Section III.A.2, by starting from the closest one. In particular Figure 9 reports, in the $Z_{3} Z_{4}$ plane, the trajectory of $S a t_{3}$, which is the one ending its stabilization phase the furthest from the target. The applied control sequence leads Sat $_{3}$ to the rendezvous condition. The approach introduced in Section III A. 2 can be easily recognized: the uncontrolled waiting phase is followed first by a positive control phase whose duration is $\Delta t^{*}$ (see Eq. (14)), then by a negative control phase of duration $2 \Delta t^{*}$ and, finally, by a positive control phase of duration $\Delta t^{*}$. The effect in the $z_{1} z_{2}$ plane is that of generating a closed symmetric trajectory (see Figure 10), as it was expected. Figure 11 shows the trajectory in $\Delta x \Delta y$ plane. The rendezvous of the other three chasers with respect to the target follow an analogous sequence of events: therefore the detailed results are omitted for the sake of brevity.


Figure 9. Simulation results: phase two, rendezvous with the target, trajectory of Sat $_{3}$ in $z_{3} z_{4}$. The plus symbol (+) indicates the initial state, the star symbol (*) indicates the final condition.


Figure 10. Simulation results: phase two, rendezvous with the target, trajectory of Sat ${ }_{3}$ in the $z_{1} z_{2}$ plane.


Figure 11. Simulation results: phase two, rendezvous with the target, trajectory of Sat ${ }_{3}$ in the $\Delta x \Delta y$ plane. The plus symbol (+) indicates the initial state, the star symbol ( ${ }^{*}$ ) indicates the final condition.

The rendezvous phase of the maneuver for $\mathrm{Sat}_{3}$ takes 1.67 hours to complete ( 1.1 orbital periods). The completion of the rendezvous phase of the maneuver for the whole fleet takes 6.16 hours ( 4.037 orbital periods). Finally, the entire maneuver, bringing the chaser spacecraft from their generic initial conditions to the rendezvous with the target, takes 18.16 hours ( 11.90 orbital periods).

For comparison purposes, the Leonard's $s^{6-7}$ approach applied to Sat $_{3}$, with the same initial conditions at the end of the stabilization phase, results in a final condition with a residual distance of 47.45 meters from the target, and a maneuver time of 18.09 hours (for the rendezvous phase only).

In order to estimate the validity of the constant air density approximation, assumed in our development, an approximate calculation for the orbit decay was performed through Eq. (18), resulting in the following variation of the semi-major axis for one orbit

$$
\begin{equation*}
\Delta a_{\text {orb }}(T)=-2 \pi\left(C_{D} S / m\right) \rho a_{o r b}^{2} \tag{18}
\end{equation*}
$$

By applying Eq. (18) to our 350 km orbit case gives a $\Delta a_{\text {orb }}=-0.44 \mathrm{~km}$. By considering this decay constant as a worst case scenario (i.e. every spacecraft has the drag plate constantly open) the altitude decay is $\sim 5.3 \mathrm{~km}$ during the 18.16 hours of the maneuver. Therefore the increase in atmospheric density appears to be negligible.

## 1. Robustness Test through Monte Carlo Simulations

In this section the results are reported of a set of Monte Carlo simulations performed to validate the robustness of the control algorithm introduced in Section III. In particular a normal distribution of relative positions and velocities has been generated with the following boundaries and 1000 simulation runs have been performed.

$$
\begin{align*}
& -3.2 \mathrm{~km}<\Delta x<3.2 \mathrm{~km} \\
& -3.2 \mathrm{~km}<\Delta y<3.2 \mathrm{~km} \\
& -0.1144 \mathrm{~m} / \mathrm{s}<\Delta \dot{x}<0.1144 \mathrm{~m} / \mathrm{s}  \tag{19}\\
& -5.54 \mathrm{~m} / \mathrm{s}<\Delta \dot{y}<5.54 \mathrm{~m} / \mathrm{s}
\end{align*}
$$

The first result of the Monte Carlo analysis consists of the confirmation of the robustness of the stabilization algorithm. In particular, during all of the simulation runs, all of the chasers were stabilized with respect to the target. The second result of the Monte Carlo analysis consists of the confirmation of the collision avoidance capability of the proposed control algorithm. In particular, in twenty of the 1000 simulation cases, the collisions were successfully avoided by adding an initial coasting phase, according to the algorithm introduced in Section III.B.3. In all of the other cases, no collisions were occurring.

The third result of the Monte Carlo analysis consists of the confirmation of the reasonable maneuvering time. In particular, Figure 12 reports the required maneuver time as a function of the initial mean distance of the chasers from the target. Figure 12 reflects an intuitively expected behavior: the total time to stabilize the whole fleet increases with the mean distance of the spacecraft from the target.

It is finally worth noting that maneuvers of longest duration (see Figure 12), lasting about 150 hours, required less than 2 minutes to be completely generated on a Pentium D 3.2 Ghz machine (corresponding to $\sim 0.02 \%$ of the maneuver duration).


Figure 12. Monte Carlo analysis: required time for completing the maneuver.

## B. Nonlinear Dynamics Simulation

In this simulation one chaser spacecraft has to rendezvous with a target which has orbital parameters similar to those of the ISS. The motion of the chaser and the target spacecraft are separately obtained by using a non-Keplerian orbital propagator. Then, their relative state vector is computed and projected in the LVLH frame, centered at any time on the target spacecraft. The relative orbit feedback stabilization control is finally used in order to drive the opening and closing of the drag plates. During the second phase of the rendezvous maneuver, the analytical algorithm presented in Section III. 2 is applied as a feedforward control. The controller is based on the linear dynamics and it still assumes a constant value of atmospheric density, as in the previous simulations (Section IV.A)

The orbital propagation for chaser and target takes into account the following effects ([17]):

1. Earth's gravitational field harmonics up to J4.
2. Variable density on both target and chaser.
3. Moon-Sun third body effects.
4. Solar Radiation Pressure.

Table 5 reports the two spacecraft orbital elements at initial time, corresponding to the following relative state in the LVLH coordinate system

$$
\left[\begin{array}{c}
x  \tag{20}\\
y \\
\dot{x} \\
\dot{y}
\end{array}\right]=\left[\begin{array}{c}
-156.4 \mathrm{~m} \\
3515.3 \mathrm{~m} \\
-0.5 \mathrm{~m} / \mathrm{s} \\
0.4 \mathrm{~m} / \mathrm{s}
\end{array}\right]
$$

Table 5: Orbital Parameters of Target and Chaser for Complete Dynamics case

| Target Spacecraft | Chaser Spacecraft |
| :---: | :---: |
| $a_{I S S}=6713889.83 \mathrm{~m}$ | $a_{I S S}=6713889.83 \mathrm{~m}$ |
| $e_{I S S}=0$ | $e_{I S S}=0$ |
| $i_{I S S}=51.94116 \mathrm{deg}$ | $i_{I S S}=51.94116 \mathrm{deg}$ |
| $\Omega_{I S S}=206.35768 \mathrm{deg}$ | $\Omega_{I S S}=206.35768 \mathrm{deg}$ |
| $\omega_{I S S}=101.07112 \mathrm{deg}$ | $\omega_{I S S}=101.07112 \mathrm{deg}$ |
| $v_{I S S}=108.08480 \mathrm{deg}$ | $v_{I S S}=108.08480+0.03 \mathrm{deg}$ |

The initial altitude of the spacecraft is approximately 336 km . The reference atmospheric density value for such altitude is reported in Table 6, together with additional characteristics of the spacecraft. The constant reference value for the atmospheric density is used within the controller. The spacecraft are assumed to be cubes with a .5 m side.

Table 6: Spacecraft Characteristics and Reference Atmospheric Density


## Figure 13. Rendezvous Trajectory for the Complete Dynamics

Figure 13 shows the rendezvous trajectory when the linear dynamics-based controller is applied to the complete dynamics. The whole maneuver takes approximately six hours to complete. The same figure also shows when the
feedforward controller takes over, after the relative orbit stabilization, and the final rendezvous error position, due to disturbances, non linear dynamics and non constant density.

Both the chaser and the target present an orbit decay of about 4 km after the maneuver is complete.
The results shown in Figure 13 indicate that the proposed control policy is robust with respect to disturbances and higher order orbital effects.

## V. Conclusion

New control logic has been introduced for the relative orbits stabilization and the subsequent rendezvous of multiple spacecraft by exploiting the differential atmospheric drag. By varying the level of aerodynamic drag of each spacecraft, relative differential accelerations are generated among the spacecraft of the group, and, therefore, their relative orbits are controlled. The proposed method can be used, for instance, for an on-orbit assembly mission.

The recently developed Schweighart-Sedwick model was used to describe the relative dynamics of different spacecraft nearby a circular orbit with the inclusion of the J2 effects. Furthermore, the natural relative dynamics of each chaser with respect to the target is decoupled into a secular motion and a periodic oscillation. In particular, the following two-phase control method was proposed. First the secular motion of each chaser is controlled via differential drag in order for the spacecraft to sequentially move from an arbitrary initial condition to a closed stable relative orbit around the target spacecraft. After the relative orbit stabilization, a relative eccentricity control is applied to each spacecraft in order to zero out the semi-axis of the relative orbit around the target and achieve the rendezvous condition.

Collisions are avoided by introducing a coasting phase before the control takes action and re-computing the whole trajectory. This is possible thanks to the analytical nature of the proposed solution, which allows for an easy and computationally light re-calculation of the whole maneuvering history.

A sample simulation was conducted by considering five spacecraft. The robustness of the stabilization control logic, collision avoidance capability and the reasonable amount of time required for the maneuver were validated through Monte Carlo analysis. In order to establish the robustness of the control logic here proposed, a two spacecraft rendezvous simulation was performed using a non-Keplerian orbital propagator. The drag control was used in a feedback fashion for the stabilization phase and as a feedforward for the rendezvous phase.

A limitation of the proposed control approach is that the final rendezvous orbit cannot be a priori specified.
The proposed methodology is applicable to a generic number of spacecraft with on-off air drag devices capabilities. The possibility of using the proposed passive orbital control for low Earth orbit spacecraft is attractive as it allows for long term propellant-free formation maneuvering.

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