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Periodic relative motion of formation flying satellites

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Abstract— In this paper the possibility to obtain natural periodic motion of formation flying is investigated through the use of a numerical global optimizer such as Genetic Algorithms. the methodology introduced has been initially verified on a case where a solution is known to exist, i.e. an unperturbed case, where the period matching condition is necessary and sufficient to have invariant relative trajectories. In the perturbed case, the conditions to obtain an invariant relative motion are known only in approximated closed forms, which, in some cases, guaranties the minimum drift, but not the relative motion periodicity. Using Genetic Algorithm we find periodic relative orbits for satellites on a J_2 perturbed orbit with two particular inclinations (63.4° for all eccentricities and 49° for nearly circular orbits); in all the other cases our method is able to supply initial conditions for minimum drift, or, in the case of orbits subjected to drag, for formations which get close after a predetermined time span.

1. INTRODUCTION

In recent years, a number of missions involving satellites in formation have been planned: a short list includes ESA missions Proba, LISA, XEUS, Darwin and SMART-3, NASA mission ST5, Air Force Research Laboratory mission TechSat21. In order to keep the satellites of the formation in the designed close configuration, and therefore to achieve mission goals, control actions are needed. The cost of this orbital control in terms of ΔV limits both the mission duration and the expected performances. A way to reduce such an amount of control action is to investigate if a suitable, natural periodic relative motion of satellites, equal or close to the desired motion, could be exploited. In such a way, drift could be almost cancelled, and a great amount of ΔV would be saved. Many works in literatures deal with this problem under different hypotheses. Inalhan, Tillerson and How (Ref. 1) find the analytical form for the initial conditions of the classical Tshauner-Hempel equations (Ref 2); Kasdin. and Koleman (Ref. 3) use the epicyclic orbital elements theory to derive bounded, periodic orbits in presence of various perturbations; Vaddi, Vadali, and Alfriend (Ref. 4) study a Hill-Clohessy-Wiltshire (HCW, Ref. 5) system modified to include second order terms; finally, Schaub and Alfriend (Ref. 6) formulate the conditions for invariant J_2 relative motion

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basing on relations between mean orbit elements of the two satellites. In all these papers, the analytical approach leads to two kinds of results: either initial conditions which ensure perfect periodicity in approximated dynamical models, or initial conditions resulting in bounded (i.e. with minimum drift, but not periodic) relative motion of more detailed dynamical models.
A numerical approach, though lacking a physical insight, guarantees a definite answer about the possibility to have periodic trajectories for satellites in a fully non linear, perturbed environment. While some results are easily predictable, like the disruptive effect of non conservative forces as atmospheric drag, some others are quite surprising and interesting. In particular, the possibility to have periodic motion is negated, as we shall see, even for a conservative, symmetric perturbation like the J_2 effect with two remarkable exceptions: in fact when the formation reference orbit is inclined at the so-called critical values (63.4° and 116.6°) and, for nearly circular orbits, at 49° and 131°, the relative motion can be considered really periodic. While the physical reasons of this behaviour are still under study, a simple conclusion can be drawn: if two satellites have to remain in close formation, the proper choice of the parameters of the reference orbit is of capital importance, and it results in a great amount of control cost saving.

2. NOTES ON GENETIC ALGORITHMS

In their simplest incarnation, genetic algorithms (GA) make use of the following reduced version of the biological evolutionary process; the gene pool - and its associated phenotypic population - evolves in response to three drivers: differential reproductive success in the population, genetic recombination (crossover) occurring at breeding and random mutations affecting a subset of breeding events.
Consider then the following generic optimization problem. Given a model (in the present case, the relative dynamics of satellites flying in formation) that depends on a set of parameters *a*, a functional relation *f(a)* returns a measure of quality for the corresponding model; this will be called *fitness function*. The optimization task consists in finding the point *a** defining the parameters of a model that maximizes the quality measure *f(a)*. The main software architecture is based on (Ref. 7).
An individual is characterized by a single chromosome, described by the unknowns of the problem:

$$a = \begin{bmatrix} x_i \\ y_i \\ z_i \\ \dot{x}_i \\ \dot{y}_i \\ \dot{z}_i \\ T_{end} \end{bmatrix} \tag{1}$$

The unknowns considered in our problem are the initial relative spatial coordinates (x_i) and the initial relative velocities ($\dot{x}_i$), which determine the formation motion. In addition, a seventh unknown is considered: the epoch T_{end} of the fitness function evaluation. This is necessary as to allow the optimizer to search also for the unknown periodicity of the relative orbit. The chosen fitness function, which establish how good an individual is, is then calculated by:

$$f(a) = \frac{1}{\sqrt{\left(\frac{x_f - x_i}{x_i}\right)^2 + \left(\frac{y_f - y_i}{y_i}\right)^2 + \left(\frac{z_f - z_i}{z_i}\right)^2 + \left(\frac{\dot{x}_f - \dot{x}_i}{\dot{x}_i}\right)^2 + \left(\frac{\dot{y}_f - \dot{y}_i}{\dot{y}_i}\right)^2 + \left(\frac{\dot{z}_f - \dot{z}_i}{\dot{z}_i}\right)^2}} \quad (2)$$

A large value of the fitness function stands to indicate that after the period T_{end} (also part of the optimisation vector), the spacecrafts have a relative state which is close to the initial relative state. To the limit, for infinite values of f(a) the mechanical deterministic principle assures us that the relative motion will repeat itself thus being invariant.

The *N* individuals of the population are initialized at random. After mating, the chromosomes of the off-springs differ from the chromosomes of the parents, because of crossover and mutation processes. The best individuals have a greater probability to mate, and so its chromosomes have a greater probability to pass their good characteristics to the off-springs. The chromosomes of the best individual after a number *M* of generations represent the solution provided by the GA.

In this kind of heuristic methods, the tuning of the algorithm is an essential and very time-demanding part of the work. The number *N* of individuals in the population, the number *M* of generations, the minimum and maximum mutation rate, the crossover probability and many other parameters can influence the results obtainable by the GA.

3. UNPERTURBED CASE

In this paragraph we analyse the unperturbed case. Here the period matching condition is the constraint that must be met in order to have a periodic motion. According to the hypotheses on the model studied, many works established the conditions for invariance; results obtained in Ref. 4 and Ref. 3 are introduced in this paper as they are used as a test case to assess GA performances.

As a first step we applied GA to the well known Hill-Clohessy-Wiltshire (HCW) equations, valid for circular unperturbed orbits:

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2 x = 0 \\ \ddot{y} + 2n\dot{x} = 0 \\ \ddot{z} + n^2 z = 0 \end{cases} \quad (3)$$

Here the condition for invariance is analytically expressed by what we call the HCW condition:

$$\dot{y}_0 = -2nx_0 \quad (4)$$

In this case GA have proved to work properly, as shown in one of our previous works (Ref. 8). Unfortunately, the condition of eq. (4) is valid for the linearized model alone.

In fact, even without considering the effects of perturbations, non linear terms neglected are more and more important as the formation dimensions grow.

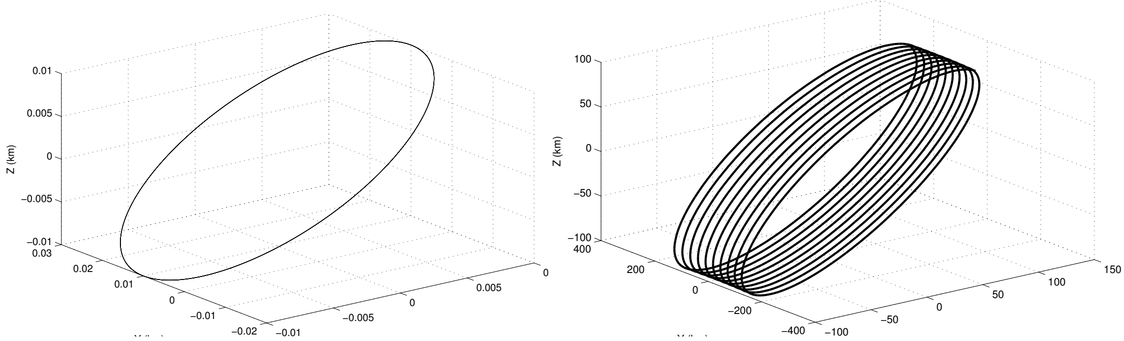

Figure 1 Relative trajectories descending from HCW condition for a small formation (left) and a large formation (right)

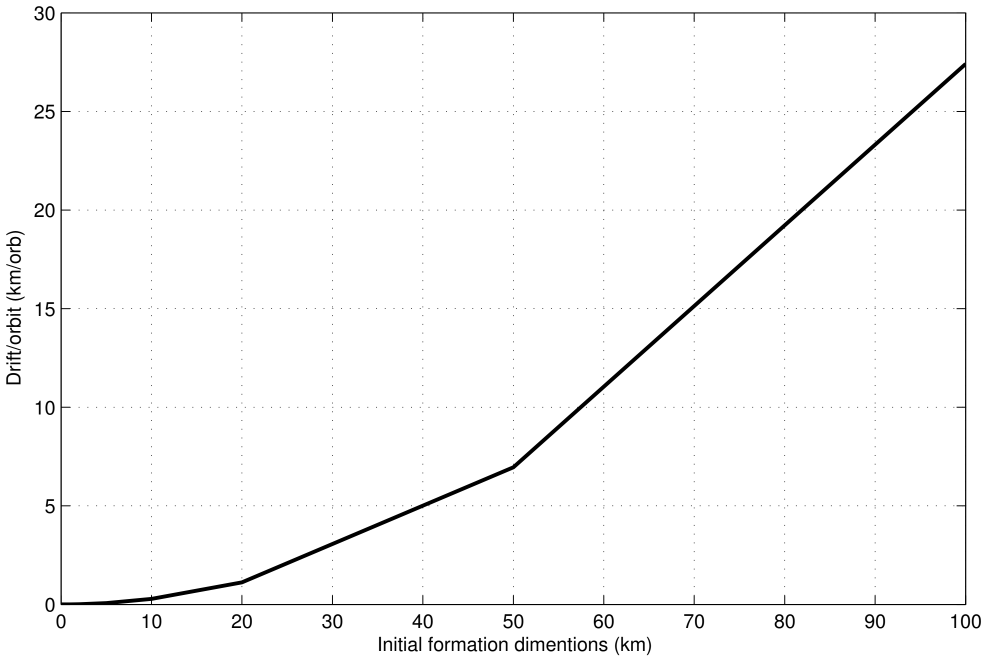

Figure 2 Field of validity of the HCW condition

Vaddi, Vadali and Alfriend (Ref. 4) have developed a model that takes into account the effects of non linearities, both for circular and for elliptic orbits.

Following the same approach of Taylor series expansion of the HCW equations, but retaining also quadratic terms, leads to the following model:

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2 x = \varepsilon\left[\dfrac{y^2}{2} + \dfrac{z^2}{2} - x^2\right] \\ \ddot{y} + 2n\dot{x} = \varepsilon xy \\ \ddot{z} + n^2 z = \varepsilon xz \end{cases} \qquad (5)$$

where $\varepsilon = \frac{3\mu}{a^4}$.

A condition for periodic relative orbits is then reached:

$$\begin{aligned}[x_0, y_0, z_0] &= [\rho/2\sin(nt+\alpha_0), \rho\cos(nt+\alpha_0), \rho\sin(nt+\alpha_0)] \\ [\dot{x}_0, \dot{y}_0, \dot{z}_0] &= [n\rho/2\cos(nt+\alpha_0), \dot{y}, n\rho\cos(nt+\alpha_0)]\end{aligned} \quad (6)$$

where ρ is the relative distance and α_0 the initial phase angle. The only variable that influence the boundedness of the relative orbit is $\dot{y}$, which can be written as:

$$\dot{y}(0) = \dot{y}_h(0) + \varepsilon\dot{y}_{cn}(0) \quad (7)$$

where $\dot{y}_h$ is the initial condition from HCW (eq. (4)) while $\dot{y}_{cn}$ is the correction for the non linearity.

$$\dot{y}_{cn}(0) = -(\rho^2/48n)(12 + 6\cos 2\alpha_0) \quad (8)$$

This is not only way to face the problem in an analytical way: in Ref. 3, Kasdin and Koleman use a Hamiltonian approach to derive the equations of motions for an object relative to a circular or slightly elliptical reference orbit. By solving the Hamilton-Jacobi equation in terms of the epicyclic elements they are able to provide analytical approximations of the invariance condition. By means of this formalism, they derive bounded, periodic orbits in the presence of various perturbations. Non-linear effects are among this perturbing actions. Here we only report the conditions found for the circular reference orbit case. Two formulas are given: one considering second-order terms in the series expansion for the initial conditions, and one considering also third-order terms.

$$a_3(0) = -\frac{5}{2}a_1^{\,2}(0) - \frac{1}{2}\left(a_2^{\,2}(0) - b_1^{\,2}(0) + b_2^{\,2}(0)\right) - 3a_1(0)b_3(0) - b_3^{\,2}(0) \quad (9)$$

$$\begin{aligned}a_3(0) = &-\frac{5}{2}a_1^{\,2}(0) - \frac{1}{2}\left(a_2^2(0) - b_1^2(0) + b_2^2(0)\right) - 3a_1(0)b_3(0) - b_3^2(0) - \frac{3}{2}(a_1^2(0)b_1(0) + \\ &+ a_2^2(0)b_1(0)) + \frac{1}{2}b_1^3(0)\end{aligned} \quad (10)$$

In both cases, it is:

$$\begin{aligned}a_1 &= \sqrt{2\alpha_1}\cos\beta_1 \\ b_1 &= \sqrt{2\alpha_1}\sin\beta_1 \\ a_2 &= \sqrt{2\alpha_2}\cos\beta_2 \\ b_2 &= \sqrt{2\alpha_2}\sin\beta_2 \\ a_3 &= \alpha_3 \\ b_3 &= \beta_3\end{aligned} \quad (11)$$

α_i and β_i are the initial canonical momenta and coordinates, which can be written as functions of the initial conditions (in the following expressions, distances are normalized by the reference orbit semi-major axis a, and the rates normalized by the angular velocity n):

$$\begin{aligned}
\alpha_1 &= \frac{1}{2}\left(\dot{x}^2 + (2\dot{y} + 3x)^2\right) \\
\alpha_2 &= \frac{1}{2}\left(\dot{z}^2 + z^2\right) \\
\alpha_3 &= \dot{y} + 2x \\
\beta_1 &= -\tan^{-1}\left(\frac{3x + 2\dot{y}}{\dot{x}}\right) \\
\beta_2 &= \tan^{-1}\left(\frac{z}{\dot{z}}\right) \\
\beta_3 &= -2\dot{x} + y
\end{aligned} \tag{12}$$

We can substitute eq. (12) in eq. (11); imposing the conditions in eq. (9) or in eq. (10) (according to the order of approximation chosen), and find $\dot{y}$ for bounded orbits.

The difference between the semi-major axes of the spacecrafts in the formation is a good index of how near the approximation of the proposed analytical conditions is to the physical one (i.e. period matching); a link between the measure of the drift per orbit and the difference in semi-major axis can be in fact expressed (Ref. 9) by:

$$-3\pi\Delta a \tag{13}$$

The difference Δa resulting by using condition (7), condition (9) or condition (10) can be plotted for various formation dimensions; as shown by Figure 3, the third-order epicyclic conditions are a very good approximation of the period matching conditions, and indeed the use of a numerical approach such as GA seems not really necessary in this case.

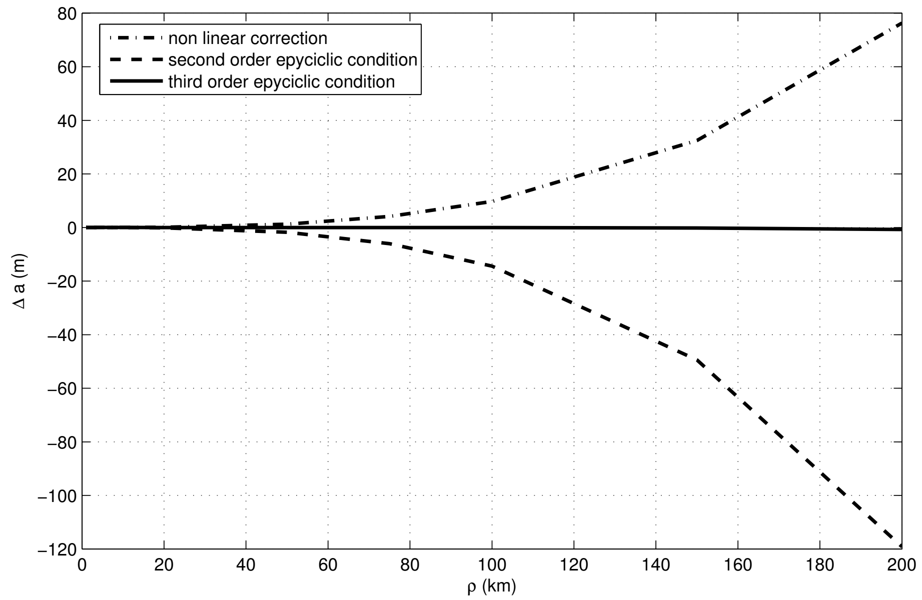

Figure 3 Difference of the semi major axis vs. initial dimensions

The comparison between analytical (third-order epicyclic) and numerical (GA) solutions is performed in Figure 4.

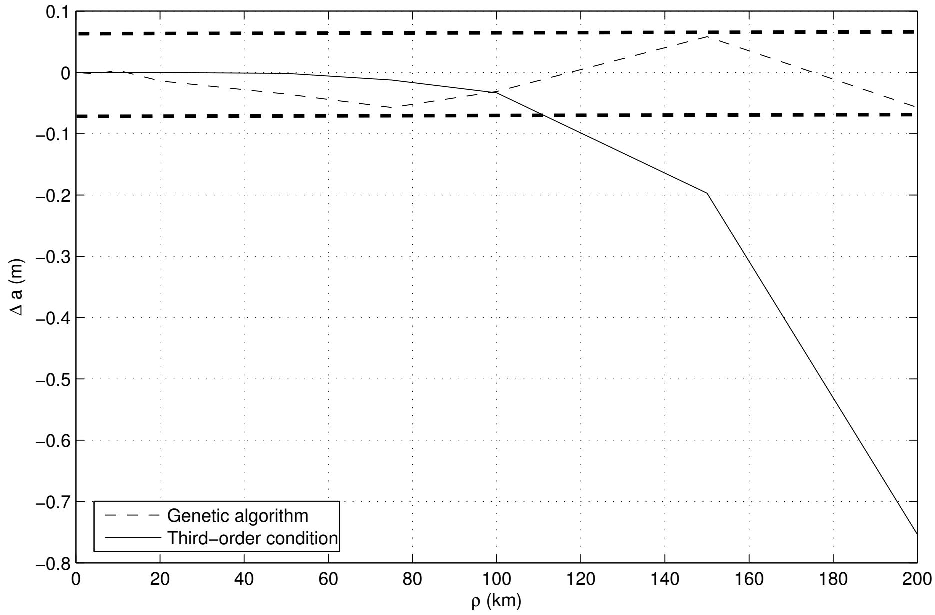

Figure 4 Comparison between GA and third-order condition

From Figure 4 it is possible to say that the main difference between analytical and numerical approach is that Δa due to genetic algorithm conditions is oscillating as it is due just to numerical errors and to the stochastic process which characterizes the optimizator; instead, Δa due to third-order conditions grows as the formation is larger. However, even for very large formations, the results of the analytical condition are quite good and the use of GA is not really necessary, but it has been now proved that when a solution exists, GA is able to find it with great accuracy.
Without any variation, the same results can be obtained for elliptical unperturbed reference orbits.

4. J_2 PERTURBED CASE

In the perturbed case the approach itself is different. The aim is not to benchmark the performances of the GA with respect to a well known closed solution, but to search if such a solution does exist or not. It is clear that a numerical approach can supply precious information that must be understood.
For low and mid height orbits J_2 effect and air drag are by far the most important perturbations.
In this paragraph J_2 perturbation is considered; the results here exposed are the most interesting that the GA approach has returned. In fact, while in the Kepler case of paragraph 3 the solution was very well known, and in the drag perturbed case of paragraph 5 the solution is easily predictable not to exist, in the J_2 perturbed case the question is open.

An analytic method is presented by Schaub and Alfriend (Ref. 6) to establish J_2 invariant relative orbits. Working with mean orbit elements, the secular drift of the longitude of the ascending node and the sum of the argument of perigee and mean anomaly are set equal between two neighbouring orbits. By having both orbits drift at equal angular rates on the average, they will not separate over time due to the J_2 influence. Two first order conditions are established between the differences in momenta elements (semi-major axis, eccentricity and inclination angle):

$$\delta a = 2Da_0\delta\eta$$
$$\delta e = \frac{\left(1-e^2\right)\tan i}{4e}\delta i \tag{14}$$

where:

$$\delta\eta = -\frac{\eta_0}{4}\tan i_0 \delta i$$
$$\eta = \sqrt{1-e^2} \tag{15}$$

and D is a parameter depending on i, a, η. Combined, eq. (14) and eq. (15) provide the two necessary conditions on the mean element differences between neighbouring orbits to yield a J_2-invariant relative orbit. When designing a relative orbit using the mean orbit element differences, either $\delta i, \delta e$ or δa is chosen, and the other two elements differences are then prescribed through the two constraints. The remaining mean orbit element differences $\delta\Omega, \delta\omega$ and δM can be chosen at will without affecting the J_2-invariant conditions. Further, note that these two conditions are not precise answers to the nonlinear problem but are only valid up to a first order approximation. Thus, relative orbits designed with these two conditions will still exhibit some small relative drift, as Figure 5 shows.

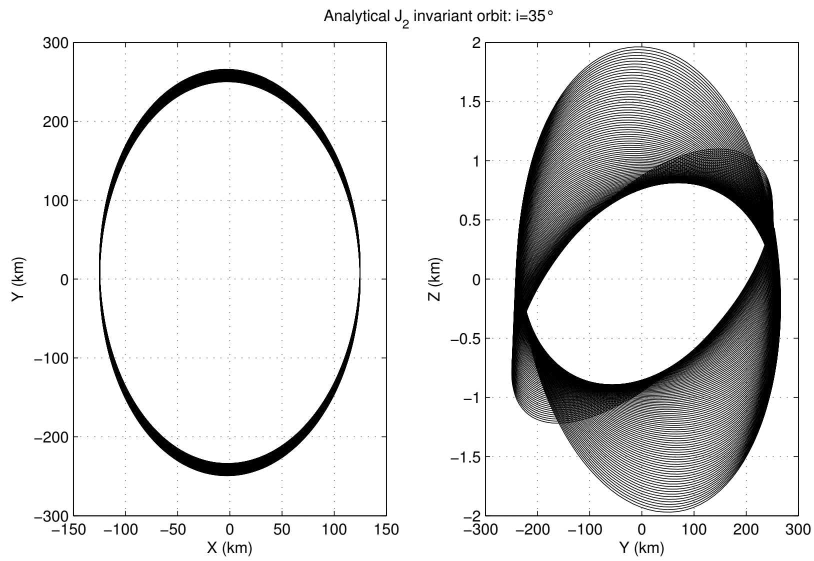

Figure 5 XY and YZ projections of relative orbits generated with analytical J2 invariant conditions (i=35°)

The conditions (14) and (15) supply two powerful means to find relative orbits which are not properly periodic but bounded, with minimum drift per orbit.
GA can be then used to verify if there is an actual physical limit for the existence of really periodic orbits, or if the residual drift is just due to the approximations accomplished when searching for analytical formulas.
A run of GA at i=35° does not improve in a remarkable way the analytical results: the drift is minimum but the periodicity is not reached. Instead, if the simulation is repeated for the entire range of inclinations, the results vary sensibly disclosing a previously overlooked feature of the invariant relative motion. Figure 6 reports the fitness function values for inclinations from 0 to 180 degrees: as told in paragraph 1, in fact, the fitness function can be seen as an index of how close to the goal the optimization has reached.

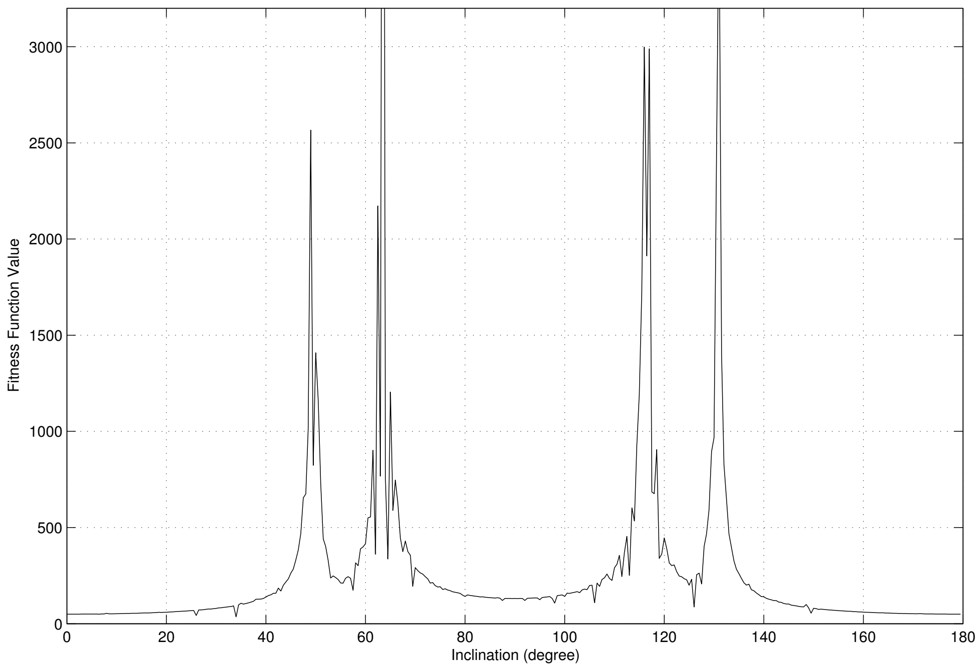

Figure 6 Fitness function values for the whole range of inclinations (nearly circular orbit)

Figure 6 shows that a value of the fitness function of the best individual ranges from 35 to 200 for formation orbiting at almost all inclinations, resulting in relative trajectories as the one illustrated in Figure 5, with two remarkable exceptions: 49° and 63.4°, and their symmetric counterparts (with respect to 90°) 131° and 116.6°.
When using GA there is always a stochastic component, so it can happen then in some simulations the fitness function for these exceptional cases is of some tens of thousands, just like in the unperturbed case (in Figure 5 y axis is limited to one thousands for clarity sake), while in some other it is much lower, depending on how lucky the initial random population is. It is anyway much higher than for ordinary inclinations.
For the critical inclinations 63.4° and 116.6°, the reason of this behaviour has to be searched in the cancellation of the mean secular drift of argument of perigee, following eq. (16).

$$\Delta\varpi = \frac{3}{2}\pi J_2 \frac{R_\oplus^{\ 2}}{p^2}\left(5\cos^2 i - 1\right) \tag{16}$$

Analyzing the Gauss' equations (17):

$$\begin{aligned}
\frac{da}{dt} &= \sqrt{\frac{p}{\mu}}\frac{2p}{\left(1-e^2\right)^2}\left[e\sin\theta\frac{f_r}{m} + \left(1+e\cos\theta\right)\frac{f_\theta}{m}\right] \\
\frac{de}{dt} &= \sqrt{\frac{p}{\mu}}\left[\sin\theta\frac{f_r}{m} + \frac{e+2e\cos\theta+e\cos^2\theta}{1+e\cos\theta}\frac{f_\theta}{m}\right] \\
\frac{d\Omega}{dt} &= \sqrt{\frac{p}{\mu}}\frac{1}{\sin i}\frac{\sin(\theta+\omega)}{1+e\cos\theta}\frac{f_z}{m} \\
\frac{d\omega}{dt} &= \sqrt{\frac{p}{\mu}}\left[-\frac{\cos\theta}{e}\frac{f_r}{m} + \frac{\left(2+e\cos\theta\right)\sin\theta}{e\left(1+e\cos\theta\right)}\frac{f_\theta}{m} - \frac{1}{\tan i}\frac{\sin(\theta+\omega)}{1+e\cos\theta}\frac{f_z}{m}\right] \\
\frac{di}{dt} &= \sqrt{\frac{p}{\mu}}\frac{\cos(\theta+\omega)}{1+e\cos\theta}\frac{f_z}{m}
\end{aligned} \tag{17}$$

it is clear that the argument of perigee is entering through the inclination i in all the equations: a growing with time of ϖ is forcing the orbital elements to have different periods: this means that the absolute motion of the single satellite is not periodic and neither can be the relative motion between the spacecrafts of the formation. Instead, at critical inclinations, a periodic absolute motion is possible, and so is a relative trajectory.

Figure 7, referring to a 100 orbits propagation, shows that at this inclination the orbit is not simply bounded, but really periodic:

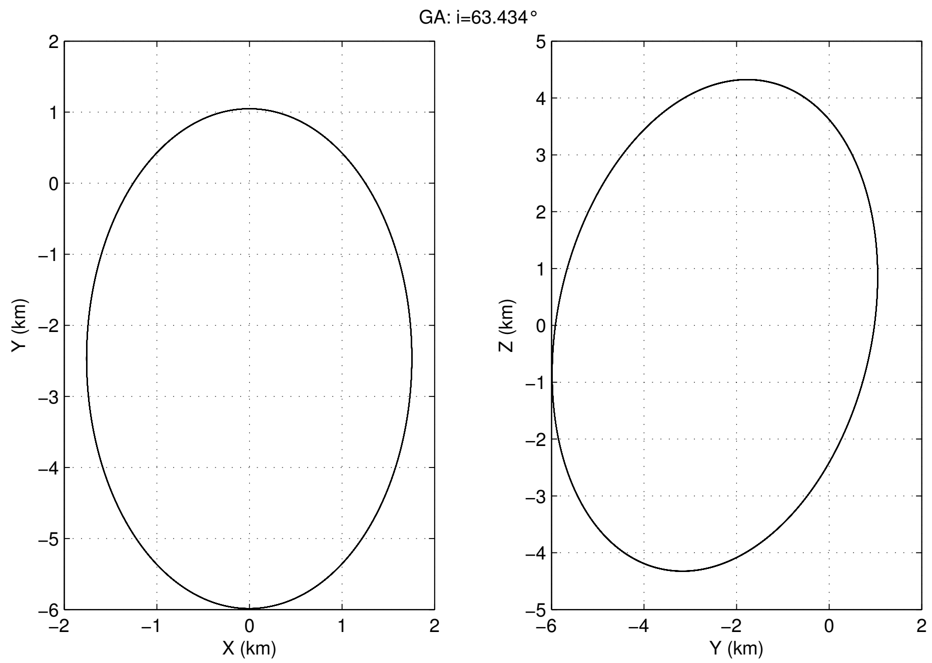

Figure 7 100 relative orbits for a J_2 perturbed case at 63.4° inclination

Though the result is very similar to the unperturbed case, here the condition is no more of period matching; in fact a difference in all the six orbital elements is kept:

	chief	deputy	Difference
a (km)	6678 km	6677.7091 km	-0.291 km
e	0.00118	0.01573	0.01455
I (degree)	63.435°	63.391°	-0.044°
ϖ (degree)	90°	50.126°	-39.87°
Ω (degree)	270°	-89.123°	0.877°
θ (degree)	0°	40.333°	40.33°
$\varpi + \theta$ (degree)	90°	90.46°	0.46°

Table 1

Table 1 refers to the case of a very large formation (that's why the differences are quite evident) and it shows how in a J_2 perturbed orbit equality of semi-major axis is not a valid condition anymore.

The second critical inclination (49°) is not yet fully understood; a further study by the same authors (Ref. 10) deals in more detail with the phenomenon. A particular interaction of the complex behaviour of the whole system seems to be at the base of it. What it seems to be the case is that this inclination is not universally valid as the critical inclinations are. In fact their validity is limited to the case of circular and nearly circular orbits, as confirmed by Figure 8, while the critical case is valid at all eccentricities. Moreover, 49° is of interest for small and middle formations, while is quite like all the other inclinations for very large formation (see Figure 9)

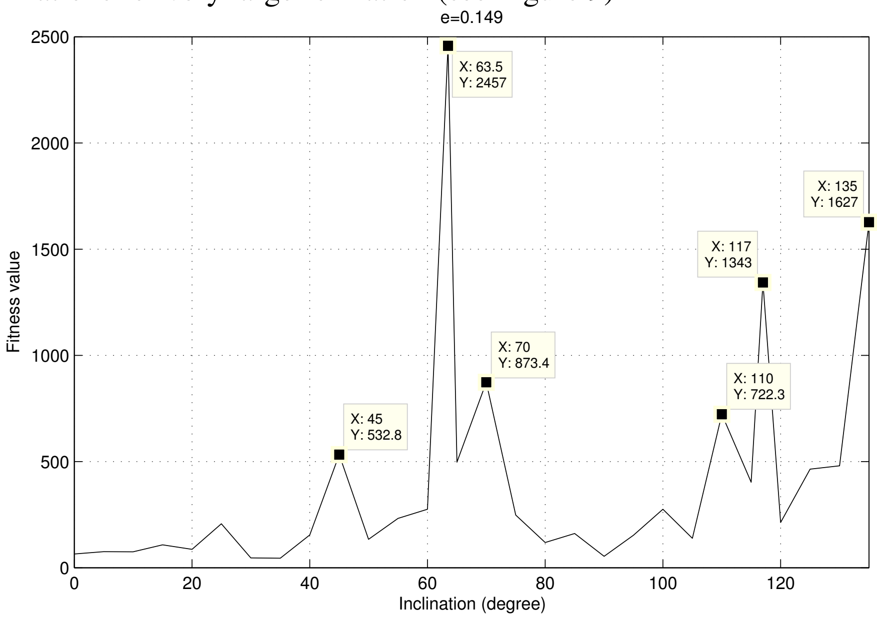

Figure 8 Fitness function values for the whole range of inclinations (elliptic reference orbit)

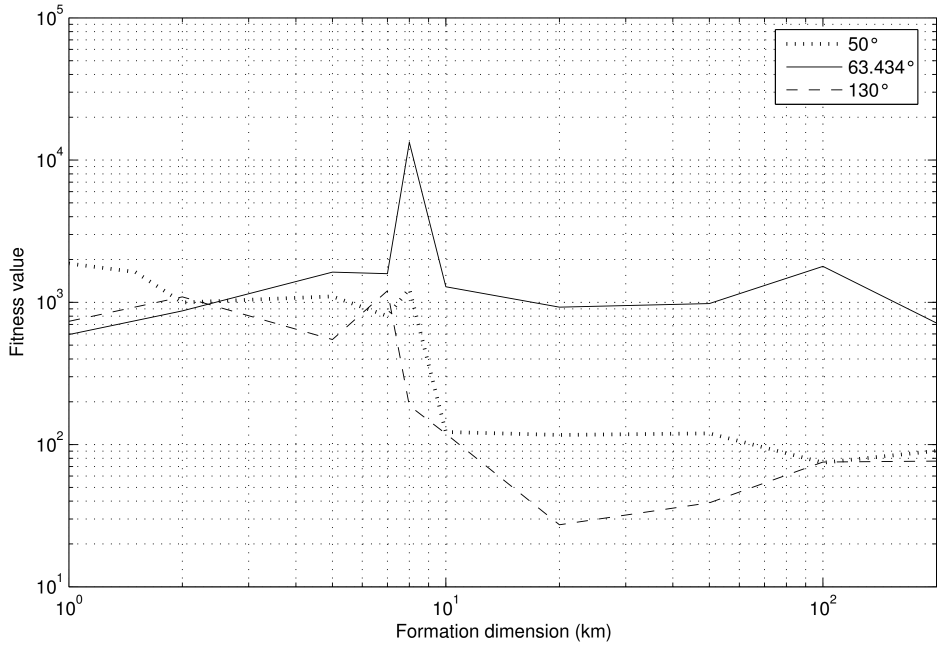

Figure 9 Fitness value as a function of the formation dimensions

Notwithstanding these limitations, the remaining field of validity is of great interest: in fact, this inclination falls in the range of inclinations of interest in the Walker's model.

5. APPLICATION OF GA ON J_2 AND DRAG PERTURBED CASE

When the orbit altitude is very low, air drag becomes of paramount importance. All the simulations performed have shown (Figure 11) what was obvious from the beginning: a dissipative perturbation such as air drag cannot allow periodic motion, even if the physical properties of the satellites (mass, area, C_D) are exactly the same, minimizing in that way the differential drag.

A different approach can be implemented in this case. Renouncing to the possibility of periodic motion, one can set GA in order to have a close formation after a predetermined number of orbits, not necessarily just 1: in this way the behaviour of the satellites between time zero and final time is considered of no interest; and the fitness function is evaluated at final time, which is the time when mission requirements ask the formation to be close.

Let's introduce an adimensional relative distance as a measure of the closeness of the formation:

$$\frac{dist}{\max(dist(\text{first orbit}))} \tag{18}$$

If the relative motion is periodic, the adimensional distance is oscillating between a minimum distance and 1. Performing the optimization with the fitness function evaluated after 1 (case A) , 50 (case B), or 100 orbits (case C), one obtains the initial conditions which, once propagated for 100 orbits, result in different dynamics: respectively, a formation which is much closer at the beginning, but then diverging (case A), or formations which seem to break apart in the first orbits, but then recomposing after at the desired time (case B and C: see Figure 11)

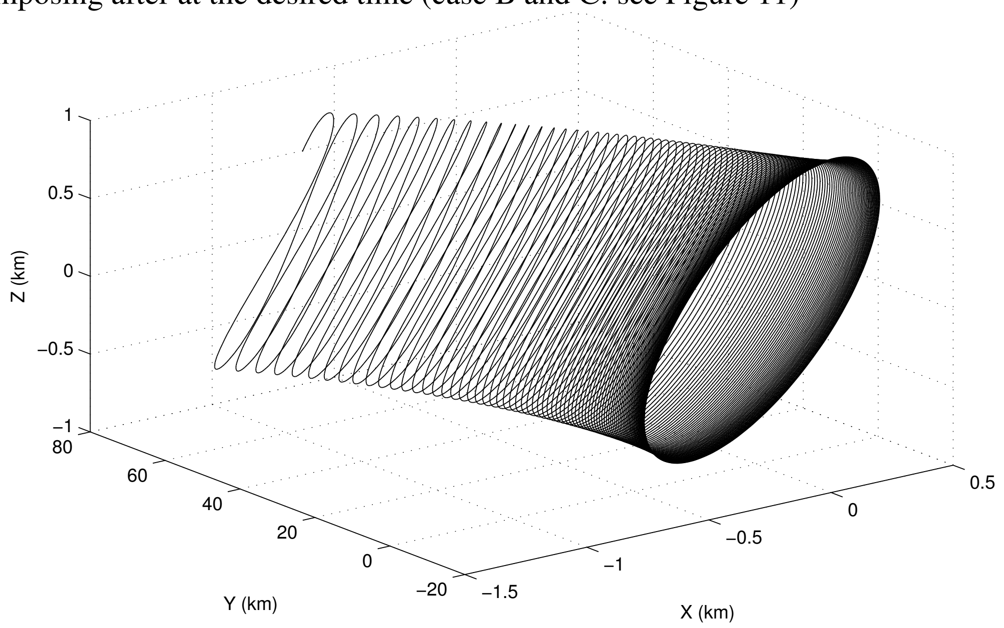

Figure 10 Best "periodic" trajectory as found by GA for a J_2 and drag perturbed orbit

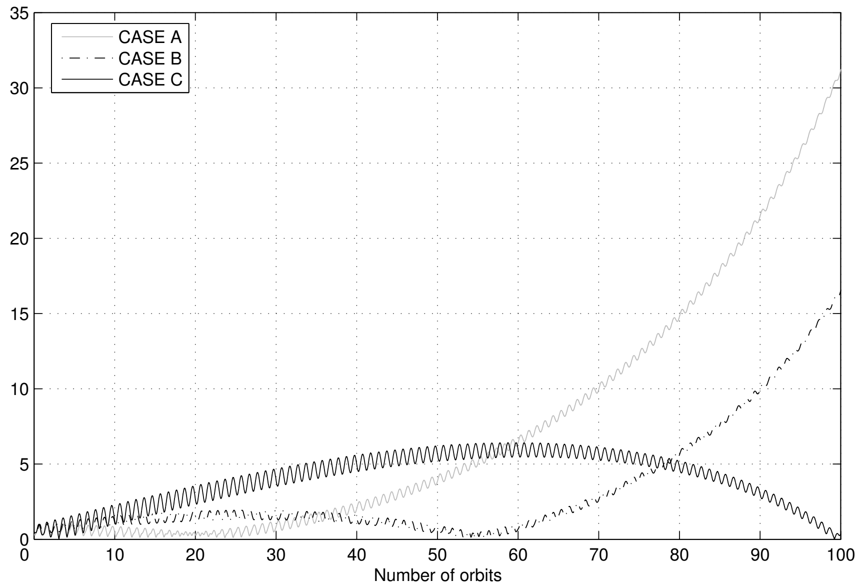

Figure 11 Comparison among adimensional relative distance for Case A (N=1), Case B (N=50) and Case C (N=100)

As an example, Figure 12 shows the behaviour in the case C: the formation seems to be breaking, but then it recomposes around the 100th orbit.

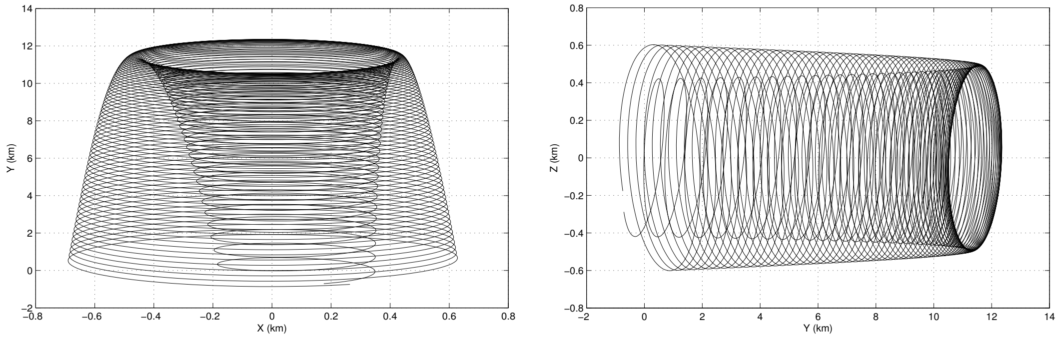

Figure 12 Projection on XY and YZ plane of the proposed strategy for the atmospheric drag effect

Other perturbations (moon-sun attraction, solar pressure) have also been analyzed, but their effect is hidden by J_2 and drag effects at low altitude, while even for high orbits like GEO, the time scale of their action is too long to be taken into account by the presented method.

6. CONCLUSIONS

The possibility to obtain natural periodic motion of formation flying satellites has been investigated through the use of a numerical global optimization technique such as Genetic Algorithms. After validating the approach by comparison to a well known test case, the unperturbed one, the attention is focused on the perturbed case. For a J_2 perturbed reference orbit GA has proved the existence of periodic relative orbits for satellites with two particular inclinations (63.4° for all eccentricities and 50° for nearly circular orbits); for drag perturbed orbits, GA supply the initial condition for a close formation after a predetermined time span (but without guarantees for the trajectory evolution before or after this time span).

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