Taking into account flexibility in attitude control[1]

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Abstract

Large structures in space do deform in a non negligible manner. The effect that deformations have on the dynamic of a spacecraft might be captured by complex non linear mathematical models. In a recent work the authors developed a non coupled set of ordinary differential equations describing the full non linear dynamic of a flexible spacecraft equipped with a system of next generation fly-wheels. These equations are here exploited to test the robustness of a "Velocity Based" steering law to flexible dynamic. As the "Velocity Based" steering law for a VSCMG device is designed under the hypothesis of rigid dynamic, "spill-over" effects are visible. Even though vibrations are suppressed quite rapidly, the "spill-over" introduces a tracking error that has to be accounted for. As a result, a minimum attitude acquisition time is found for a selected satellite configuration, faster manoeuvres are not possible as flexible modes would be excited in such a way as to make the spacecraft lose the required pointing accuracy.

Introduction

In recent years satellite platforms have increased their performances in attitude maneuverability and precision. Among the attitude actuators flywheels are certainly the most used for precision pointing. These wheels store angular momentum and exchange it with the satellite platform causing the spacecraft to reorient. Control Moment Gyros (CMG) and Variable Speed Control Moment Gyros (VSCMG) are such devices that have been recently given a fair attention due to their capability of exerting greater torques. The problem of designing a control steering law for a spacecraft equipped with these modern attitude actuators has been faced by several authors. A VSCMG device is a wheel gimballed to the spacecraft main structure that has the ability to rotate around the gimbal axis as well as chaniging its wheel speed. These control devices allow to avoid the singular configurations that affect CMG. When a singular configuration approaches, a properly chosen spin axis angular velocity variation may in-fact still generate the required torque. The problem of tracking a given

[1]This work is dedicated to the memory of Prof. Chiara Valente.

attitude history using these devices has been faced and solved in some recent works by Shaub et al. [1], Vadali et al. [2] and Tsiotras et al. [3] using a Lyapunov approach and considering the gimbal and wheel speed as controls. In these articles a "Velocity Based" steering law is developed that exploit in an optimal way the redundant degrees of freedom available with CMG and VSCMG. In this work, the equations describing the motion of a multi-flexible body, mainly taken from [4] and [5], are used to test the robustness of this velocity based steering law to flexible dynamic.

The steering law

In the works [1, 3, 2] the problem of attitude and power tracking for satellites equipped with Control Moment Gyros or variable Speed Control Moment Gyros is faced. By a clever handling of Lyapunov theory the problem is there solved leading to a steering law based upon the rigid body dynamic equations:

$$\begin{aligned} &\mathbf{J}_T\dot{\omega}+\textstyle\sum_k Y_{g_k}\ddot{\gamma}_k\mathbf{g}_k+\sum_k I^w_{s_k}\dot{\Omega}_k\mathbf{s}_k+\sum_k I^w_{s_k}\Omega_k\dot{\gamma}_k\mathbf{t}_k+ \\ &+\textstyle\sum_k(Y_{s_k}-Y_{t_k})\dot{\gamma}_k(\mathbf{t}_k\mathbf{s}_k^T+\mathbf{s}_k\mathbf{t}_k^T)\omega+\omega^x\mathbf{J}_T\omega+ \\ &+\textstyle\sum_k Y_{g_k}\dot{\gamma}_k\omega^x\mathbf{g}_k+\sum_k I^w_{s_k}\Omega_k\omega^x\mathbf{s}_k=\mathbf{g} \end{aligned} \tag{1}$$

where ω is the spacecraft angular velocity projected in body axes, γ_j are the angular positions of the n gimbal plus wheel devices and Ω_j are the wheels spin velocities. The above equations, equivalent to those used in [1, 3, 2], may be derived from [5, 4] by neglecting the flexible terms. We here briefly recall the developments of the above mentioned steering law.

Kinematics relations for the whole spacecraft given in terms of Modified Rodriguez Parameters (MRP) have to be added to Eq. (1) in order to have a complete set of motion equations. Let us introduce ω_d and σ_d respectively as the tracking signal for the angular velocity and for the MRP, equation (1) together with the kinematics relation can be written in the compact form:

$$\mathbf{f}(\dot{\mathbf{x}},\mathbf{x},\mathbf{u})=0$$

where $\mathbf{x}=[\omega_e,\sigma_e]$, $\mathbf{u}=[\dot{\gamma}_j,\dot{\Omega}_j]$, $\omega_e=\omega-\omega_d$ and σ_e is the MRP that overlaps the actual body frame to the desired one. Following the development in [3] a Lyapunov function $V(\omega_e,\sigma_e)$ may be introduced and the following relation may be obtained by imposing that V is negative definite during the spacecraft motion:

$$\mathbf{B}\ddot{\gamma}+\mathbf{C}\dot{\gamma}+\mathbf{D}\dot{\Omega}=\mathbf{L}_{rm} \tag{2}$$

where $\mathbf{L}_{rm}$, $\mathbf{B}$, $\mathbf{C}$ and $\mathbf{D}$ are functions of the tracking signal, of the state and of the gimbal angles and the wheels speed. Eq. (2) may be regarded to as a set of 3 relations between the $2n$ unknown quantities $\dot{\gamma}_j,\dot{\Omega}_j$. By neglecting the term in $\ddot{\gamma}$ (excessive gimbal acceleration are carefully avoided as result in infeasible steering laws) Eq.(2) can be rewritten as

$$\mathbf{Q}\mathbf{u}=\mathbf{L}_{rm}. \tag{3}$$

As these equations are not sufficient to determine all of our unknowns as soon as $n > 1$, a selection between all the controls time histories that make V negative definite has to be made. This might be done in several ways reminding that gimbal accelerations have to be kept small and that singular configurations have to be avoided. The result is a “Velocity Based” steering law (the name reminds us that gimbal accelerations have been neglected), capable (in theory) to perfectly track any given signal. It has however to be noted that, as soon as the signal becomes too demanding, the spacecraft may excite its flexible modes and its dynamic may therefore part from a rigid one. This may lead to an inaccurate tracking and to undesirable effect. The rest of this paper is focused on this last issue. Equation of flexible dynamic are taken from [5] and [4], these equations have been developed by taking care to follow both the methods dealing with multibody dynamic and those dealing with VSCMG dynamic. As some structural invariants are involved in this general formulation, the following section will select and define a test case and will evaluate such invariants.

Spacecraft’s test configuration

A simple geometrical configuration has been used in order to obtain the necessary structural invariants by means of analytical formulas (a preliminary FEM analysis is otherwise necessary whenever the structure considered is complex). Let us consider a spacecraft with a 6m long flexible boom mounted as shown in figure 1. A tip mass is also considered. The body frame $\mathcal{F}_b$ is centered in the center of mass G of the undeformed satellite and has the x axis aligned with the undeformed boom. The boom root has a distance of .6 meters from the point G. Considering the standard modal analysis for a cantilever beam with tip mass the following normal modes may be evaluated:

$$\phi_i(x) = \frac{1}{c_i\sqrt{L}}\left\{\cosh\left(\frac{\lambda_i x}{L}\right) - \cos\left(\frac{\lambda_i x}{L}\right) - B_i\left[\sinh\left(\frac{\lambda_i x}{L}\right) - \sin\left(\frac{\lambda_i x}{L}\right)\right]\right\}$$

with

$$B_i = \frac{\sinh(\lambda_i L) - \sin(\lambda_i L)}{\cosh(\lambda_i L) + \cos(\lambda_i L)}$$

The eigenvalues λ_i are the solutions of the equation:

$$1 + \cos(\lambda_i L)\cosh(\lambda_i L) - \frac{m_{tip}}{m_{beam}}\lambda_i L\left(\sin(\lambda_i L)\cosh(\lambda_i L) - \cos(\lambda_i L)\sinh(\lambda_i L)\right) = 0$$

For a selected ratio of the tip mass to the beam mass of .75 the first eigenvalue is $\lambda_1 = 1.32$ corresponding to a normalization constant $c_1 = .47$. Hence we may derive the first two shape functions (one for each direction) in our case:

$$\begin{aligned}\phi_1 &= [0, \phi_1, 0]^T \\ \phi_2 &= [0, 0, \phi_1]^T\end{aligned}$$

By assuming as flexible stiffness $EI = 12Nm^2$, the translational and rotational participation factors may be evaluated, together with the matrices $\mathbf{G}_i$, $\mathbf{H}_{ij}$, $\mathbf{K}$ and the

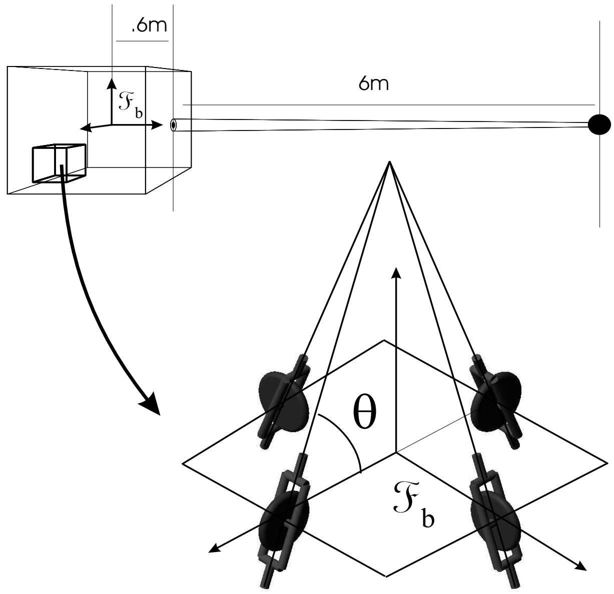

Figure 1: Geometrical configuration considered

vectors $\mathbf{P}_{ij}$ that appear in the equations used (see [4]). The linear density of the boom has been set as $\rho = .3667 kg/m$. The natural frequency of the system resulted to be $\omega_1 = \omega_2 = .65 rad/sec$. The damping matrix $\mathbf{C}$ has been set to be $\mathbf{C} = \mathrm{diag}[.13, .13] 10^{-3} N \cdot m \cdot s^2$.

The matrix $\mathbf{J}$ comprehensive of the VSCMG point masses inertias has been set to be

$$\mathbf{J} = \begin{pmatrix} 22.9 & 6.4 & 7.6 \\ 6.4 & 128.6 & 5.1 \\ 7.6 & 5.1 & 128.6 \end{pmatrix}$$

As a final remark we underline that, from a numerical point of view, there is no difference between the simple geometry here analyzed and a more complex case, as the sole numerical values of the structural invariants would be different.

Numerical results

An angular acquisition manoeuvre is here simulated. A satellite platform is required to change its orientation acquiring the desired final attitude in a given time . Two different cases will be simulated in order to verify the “Velocity Based” control law robustness during maneuvers in which actuators have to generate torques of different magnitude. The desired MRP profiles are given in figure (2) for both maneuvers. The rotation is performed with respect to an axis of coordinates $[\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}]^T$ in the body axis reference frame.

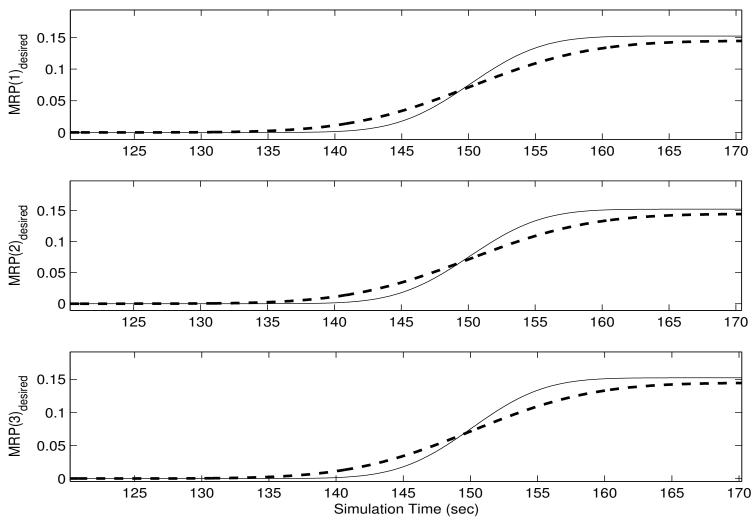

Figure 2: Desired MRP for an angular acquisition maneuver.

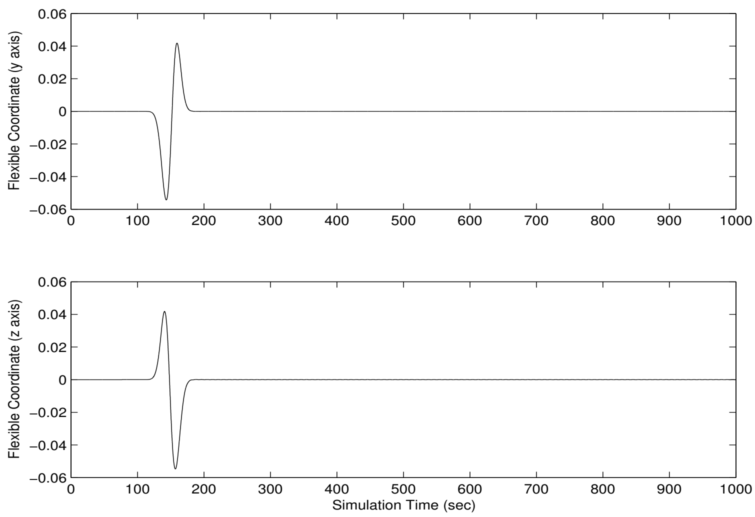

Figure 3: Flexible coordinates for the first simulation.

Both manoeuvres acquire the same attitude but with different speed. The flexible coordinates during these manoeuvre are shown in figures (3) and (4). As it is easily seen from these figures, the control law succeeds in reducing the structural vibrations acting on ω and σ. In both the graphs a main oscillation is present due to the system reply to the signal angular acceleration. When the signal angular acceleration disappear, residual vibrations with a decreasing amplitude may be noted. The damping

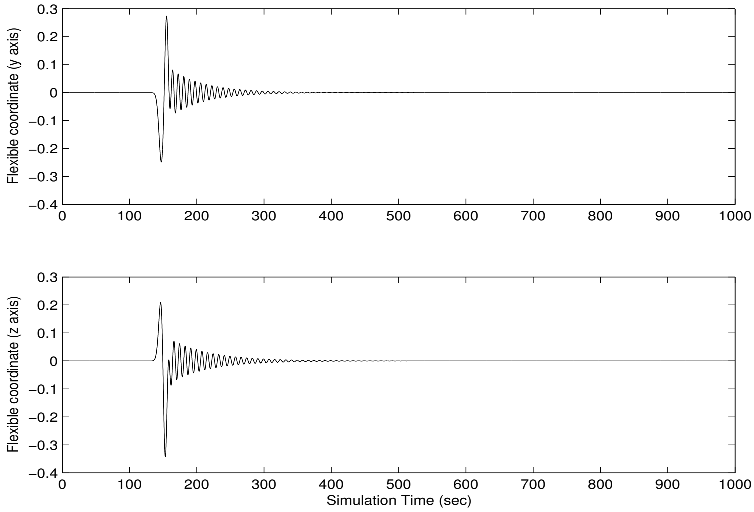

Figure 4: Flexible coordinates for the second simulation.

rate of these vibrations is greater than the natural one and it is due to the steering law detecting tracking errors and trying to correct them. During the first simulation the maximum tip deflection was of about 6*cm*, while the second simulation returned a maximum tip bending of about 30*cm*. The errors related to the two different maneuvers have been compared in terms of the MRP. As shown in figure (5) for the first maneuver (the dotted one in figure 2) the difference between the spacecraft MRP and the desired one is of an order of (10^{-4}) after 200*sec*. In this simulation the tracking signal does not excite structural vibrations and the errors displayed in figure (5) are not due to residual oscillations. On the other hand the tracking signal in the second maneuver is able to excite vibrations as can be seen in figure (6). The effect due to structural dynamic causes the acquisition to be slower even if the tracking signal is faster. Thus, after 200*sec*, the difference between MRP and MRP desired is still about 10^{-3}, one order of magnitude greater than in the previous simulation. An accurate pointing budget has to take into account this coupling between flexible vibrations and attitude dynamic. In order to accomplish the acquisition in the shortest possible time the tracking signal has therefore to be carefully selected. If an angular tolerance of 0.1° is required, the acquisition time may be evaluated for increasingly faster signals. In figure (7) real angular acquisition times against the nominal tracking signal acquisition time are plotted for several input guidance shapes. The figure clearly shows how, by taking into account flexible dynamic, fast tracking signals lead to increasing flexible dynamic effects so that the required pointing accuracy is reached in a greater time.

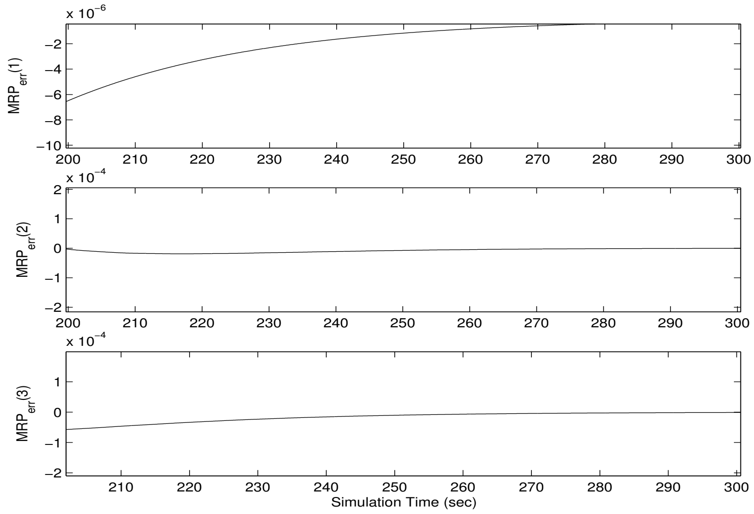

Figure 5: σ_e for the first angular acquisition maneuver between 200*sec* and 300*sec*.

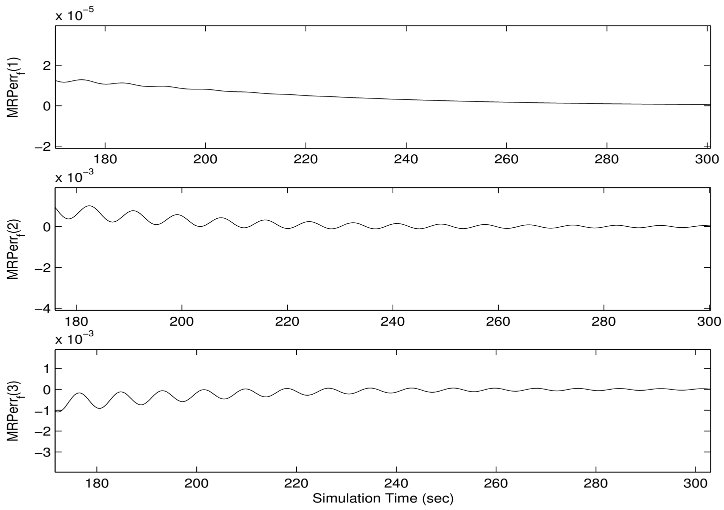

Figure 6: σ_e for the second angular acquisition maneuver between 200*sec* and 300*sec*.

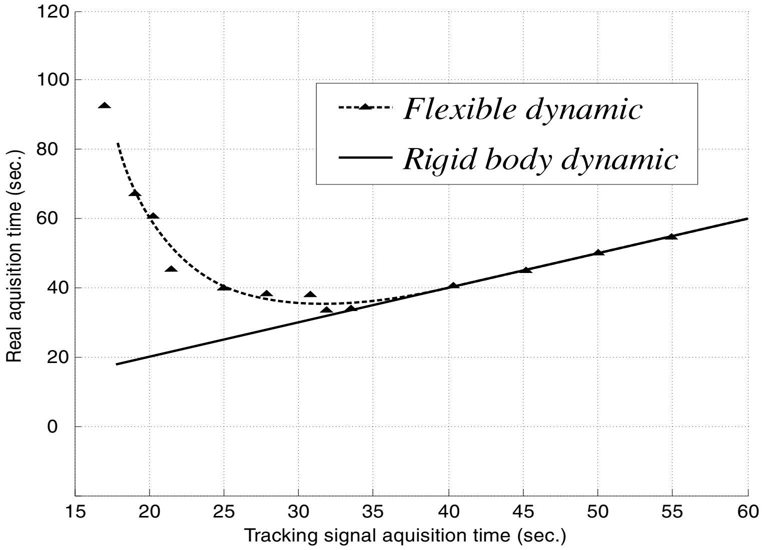

Figure 7: Acquisition desired time against acquisition real time.

Conclusions

A control steering law designed for a rigid spacecraft equipped with Variable Speed Control Moment Gyros is simulated in a flexible dynamic case. The influence of structural vibrations on the steering law performance has been verified in increasingly more demanding cases. Two different maneuvers of angular acquisition have reported as an example. The two maneuvers realize the same angular acquisition in different times. Even though the control law manages to reduce the vibration amplitude by acting on ω and $\mathbf{q}$, flexible dynamics influence the final acquisition time. For each acquisition manoeuvre an optimal tracking signal that realizes the acquisition within the required precision and in a minimum time is found.

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